

Theoretical Results for Fully Flooded, Elliptical Hydrodynamic Contacts

Bernard J. Hamrock
*Lewis Research Center
Cleveland, Ohio*

and

Duncan Dowson
*The University of Leeds
Leeds, England*

October 1982

NASA

LIBRARY COPY

1982 OCT 20

LESLIE CRISTIANI - 100-101
100-101-101A
PENNICK, JENNIFER

DISPLAY 83N13458/2

83N13458*# ISSUE 4 PAGE 526 CATEGORY 37 RPT#: NASA-TM-81696 E-203
NAS 1.15:81696 82/10/00 84 PAGES UNCLASSIFIED DOCUMENT

UTTL: Isothermal elastohydrodynamic lubrication of point contacts

AUTH: A/HAMROCK, B. J.: B/DOWSON, D. PAA: B/(Leeds Univ., Engl.)

CORP: National Aeronautics and Space Administration, Lewis Research Center,
Cleveland, Ohio. AVAIL.NTIS SAP: HC A05/MF A01

Repr. from Ball Bearing Lubrication, Sep. 1981.

MAJS: /*BALL BEARINGS/*ELASTOHYDRODYNAMICS/*ELLIPTICITY/*FILM THICKNESS/*
LUBRICATION

MINS: / BRONZES/ INTERFEROMETRY/ LUBRICANTS/ PLOTTING/ PRESSURE DISTRIBUTION/
SILICON NITRIDES/ STEELS

ABA: Author

ABS: The influence of the ellipticity parameter and the dimensionless speed,
load, and materials parameters on minimum film thickness was investigated.
The ellipticity parameter was varied from 1 (a ball-on-plate
configuration) to 8 (a configuration approaching a line contact). The
dimensionless speed parameter was varied over a range of nearly two orders
of magnitude. Conditions corresponding to the use of solid materials of
bronze, steel, and silicon nitride and lubricants of praffinic and
naphthemic mineral oils were considered in obtaining the exponent in the
dimensionless materials parameter. Thirty-four different cases were used
in obtaining the minimum film thickness formula $H_{min} = 3.63U$ to the 0.68

MORE ENTER:

DISPLAY 83N13458/2

to the 0.49 power W to the -0.073 power $1-e$ to the 0.68K power). A
simplified expression for the ellipticity parameter was found where $k =$
1.03 $(r(y)/r(x))$ to the 0.64 power. Contour plots were also shown which
indicate in detail the pressure spike and two side lobes in which the
minimum film thickness occurs. These theoretical solutions of film
thickness have all the essential features of the previously reported
experimental observations based upon optical interferometry.

ENTER:

*A DATA Change HAS Been made to correct
the title on DATA BASE*

Theoretical Results for Fully Flooded, Elliptical Hydrodynamic Contacts

Bernard J. Hamrock
*Lewis Research Center
Cleveland, Ohio*

and

Duncan Dowson
*The University of Leeds
Leeds, England*



National Aeronautics and
Space Administration

Lewis Research Center

1982

N83-13458#

THEORETICAL RESULTS FOR FULLY FLOODED, ELLIPTICAL HYDRODYNAMIC CONTACTS

The most important practical aspect of the elastohydrodynamic lubrication (EHL) theory developed in the last chapter is the determination of the minimum film thickness within the contact. The maintenance of a fluid film of adequate magnitude is an essential feature for the correct operation of lubricated machine elements. The results for a fully flooded conjunction are given in this chapter and are based on the earlier work of Hamrock and Dowson (1977a). These results show the influence of contact geometry on minimum film thickness as expressed by the ellipticity parameter and the dimensionless speed, load, and materials parameters.

In the numerical analysis the ellipticity parameter was varied from 1, which represents two spheres in contact or a ball on a plane, to 8, which represents a long ellipse and a conjunction geometry approaching that of two cylinders or a cylinder on a plane in line contact. The dimensionless speed and load parameters were varied over about two and one order of magnitude, respectively. Conditions equivalent to using solid materials of

*Published as Chapter 8 in Ball Bearing Lubrication by
Bernard J. Hamrock and Duncan Dowson, John Wiley & Sons, Inc.,
Sept. 1981.

bronze, steel, and silicon nitride and lubricants of paraffinic and naphthenic mineral oils were also considered in deriving numerical solutions and hence in obtaining the exponent on the dimensionless materials parameter. Thirty-four cases were considered in obtaining the fully flooded minimum- and central-film-thickness formulas. Experimental verification of these relationships is presented in Chapter 10.

A fully flooded condition is said to exist when the inlet distance of the conjunction ceases to influence in any significant way the computed minimum film thickness. The appropriate inlet dimension for the conjunction is defined as the distance from the center of the Hertzian contact zone to the inlet edge of the computing area.

In this chapter contour plots of both pressure and film thickness are presented that show the essential features of elastohydrodynamic conjunctions. In particular the crescent-shaped region of minimum film thickness, with its side lobes in which the separation between the solids is a minimum, clearly emerges in the numerical solutions. These theoretical solutions for film thickness have all the essential features of previously reported experimental observations based on optical interferometry that are presented in Chapter 10.

8.1 Dimensionless Grouping

The variables resulting from the isothermal EHL elliptical-contact theory developed in Chapter 7 are

A, B = constants used to define density of fluid, m^2/N

E' = effective elastic modulus, N/m^2

F = normal applied load, N

h = film thickness, m

$P_{iv,as}$ = asymptotic isoviscous pressure obtained from Roelands (1966), N/m^2

R_x = effective radius in x (motion) direction, m

R_y = effective radius in y (transverse) direction, m

u = surface velocity in x direction, m/sec

Z_1 = viscosity-pressure index, a dimensionless constant

η_0 = atmospheric viscosity, $\text{N s}/\text{m}^2$

Dowson and Higginson (1966) found that changes in the lubricant density with pressure had little effect on the minimum film thickness for line or rectangular contacts. We may therefore assume that the same is true for elliptical contacts. Even though the compressibility effect is still considered in the EHL theory developed in Chapter 7, the constants used to define the fluid in the density equation are not used in the minimum-film-thickness formula. Therefore the 11 variables mentioned previously are reduced to nine, A and B being eliminated. From the

nine variables the following five dimensionless groupings were established:

(1) Dimensionless film thickness

$$H = \frac{h}{R_x} \quad (8.1)$$

where

$$\frac{1}{R_x} = \frac{1}{r_{ax}} + \frac{1}{r_{bx}} \quad (8.2)$$

The radii of curvature in equation (8.2) are shown in Figure 2.18.

(2) Dimensionless load parameter

$$W = \frac{F}{E' R_x^2} \quad (8.3)$$

where

$$E' = \frac{2}{\frac{1 - \nu_a^2}{E_a} + \frac{1 - \nu_b^2}{E_b}} \quad (8.4)$$

(3) Dimensionless speed parameter

$$U = \frac{\eta_0 u}{E' R_x} \quad (8.5)$$

where

$$u = \frac{u_a + u_b}{2} \quad (8.6)$$

(4) Dimensionless materials parameter

$$G = \frac{E'}{P_{iv,as}} \quad (8.7)$$

where $P_{iv,as}$ is the asymptotic isoviscous pressure obtained from Roelands (1966). Blok (1965) has pointed out that the

asymptotic isoviscous pressure can be approximated by the inverse of the pressure-viscosity coefficient reported by Roelands (1966); thus $p_{iv,as} \approx 1/\alpha$.

(5) Ellipticity parameter

$$k = \frac{a}{b} \quad (8.8)$$

where

a = semiaxis in the y (transverse) direction

b = semiaxis in the x (motion) direction

The ellipticity parameter is determined from the definition of the radii of curvature (r_{ax} , r_{bx} , r_{ay} , and r_{by}); the derivation is given in Chapter 3.

The dimensionless film thickness can thus be written as a function of the other four parameters

$$H = f(k, U, W, G) \quad (8.9)$$

In the fully flooded results presented in this chapter the influence of the dimensionless parameters k , U , W , and G on minimum film thickness is studied.

The set of dimensionless groups $\{H, U, W, G, \text{ and } k\}$ is a useful collection of parameters for evaluating the results presented in this and the subsequent chapter. It is also comparable to the set of dimensionless parameters used in the initial elastohydrodynamic analysis of line contacts, and it has the merit that the physical significance of each term is readily apparent. However, a number of authors, for example, Moes

(1965-66) and Theyse (1966), have noted that this set of dimensionless groups can be reduced by one parameter without any loss of generality. This approach is followed in Chapter 12, where the film thicknesses to be expected in each of the four regimes of fluid-film lubrication are represented graphically.

8.2 Effect of Ellipticity Parameter

The ellipticity parameter k is a function of the radii of curvature of the solids only (r_{ax} , r_{bx} , r_{ay} , and r_{by}). The radii of curvature in the x -direction for both solids a and b are used in defining the dimensionless speed and load parameters. Therefore only the radius of curvature of solid b in the y direction was changed in varying the ellipticity parameter. In doing this the dimensionless parameters for speed U , load W , and materials G were held constant at the following values:

$$U = 0.1683 \times 10^{-11}, W = 0.1106 \times 10^{-6}, G = 4522 \quad (8.10)$$

Care was taken to ensure that the highest value of the ellipticity parameter ($k = 8$) corresponded to conditions in the fully developed elastohydrodynamic region. The approach used was to relate the operating conditions for this near-rectangular-contact situation to the elastohydrodynamic region defined by Dowson and Whitaker (1965-66). For values of k less than 8 the results move further into the elastic region.

Table 8.1 shows the computed values of minimum film thickness H_{\min} for 10 values of the ellipticity parameter as obtained from the EHL elliptical-contact theory developed in Chapter 7. From these 10 pairs of data it was possible to determine a good empirical relationship between the minimum film thickness H_{\min} and the ellipticity parameter k for the conditions considered in the computations.

The form of relationship chosen after investigating a number of different expressions can be written as

$$1 - \frac{H_{\min}}{H_{\min,r}} = \tilde{A} e^{\tilde{B}k} \quad (8.11)$$

A least-squares exponential curve was fitted to the 10 pairs of data points to obtain values for \tilde{A} and \tilde{B} in equation (8.11).

$$\left[k_i, \left(1 - \frac{H_{\min}}{H_{\min,r}} \right)_i \right], \quad i = 1, \dots, 10$$

In addition to the least-squares fit a coefficient of determination ϵ was obtained. The value of ϵ reflected the fit of the data to the resulting equation: unity representing a perfect fit, and zero the worst possible fit. The minimum film thickness for a rectangular contact $H_{\min,r}$ used in equation (8.11) was determined by finding the value of $H_{\min,r}$ that gave a coefficient of determination closest to unity. The rectangular-contact minimum film thickness was thus deduced from

the present set of results for the limiting case in which the ellipticity parameter k approached infinity. This value of $H_{\min,r}$ turned out to be 7.082×10^{-6} with a corresponding coefficient of determination of 0.9990, an excellent fit. The values of \tilde{A} and \tilde{B} in equation (8.11) obtained from the least-squares fit were $\tilde{A} = 0.9966 \approx 1.00$ and $\tilde{B} = -0.6752 \approx -0.68$.

Substituting these values of \tilde{A} and \tilde{B} into (8.11) established the following relationship between the ellipticity parameter and the minimum film thickness:

$$\tilde{H}_{\min} \approx (1 - e^{-0.68k}) \quad (8.12)$$

where $\tilde{H}_{\min} = h_{\min}/R_x$ represents the value of dimensionless minimum film thickness predicted by the relationship that gave the best least-squares fit to the numerical solutions. It is most significant that the computed value of \tilde{A} was 0.9966, or approximately unity, since this ensured that the minimum film thickness approached zero as k approached zero. Therefore, even though the smallest value of k used in obtaining equation (8.12) was unity, it appears that equation (8.12) might well be valid at smaller values of k since in the limiting case where equation $k \rightarrow 0$ equation (8.12) satisfies the physical situation corresponding to a long elliptical line contact in the direction of motion. For the other extreme case of a large value of k a nominal line or rectangular-contact situation was approached, and the agreement with existing results was again good. From Dowson and Higginson (1966) the rectangular-contact minimum film

thickness for the dimensionless parameters given in equation (8.10) was 7.720×10^{-6} , compared with 7.082×10^{-6} from the present results. The difference of only 9 percent could well be the result of Dowson and Higginson (1966) using an exponential pressure-viscosity relationship instead of the Roelands (1966) formula used in the present work. As pointed out in the closure of Hamrock and Dowson (1976b), in answer to a query from the late P. M. Ku, the Roelands formula for viscosity as a function of pressure suppresses the pressure spike somewhat and results in a smaller film thickness than that predicted for an exponential viscosity-pressure relationship.

The values of \tilde{H}_{\min} for the 10 values of the ellipticity parameter considered are compared with H_{\min} in Table 8.1. The percentage difference between the minimum film thickness obtained from the EHL elliptical-contact theory H_{\min} and the minimum film thickness from the least-squares-fit equation \tilde{H}_{\min} is expressed as

$$E_1 = \left(\frac{\tilde{H}_{\min} - H_{\min}}{H_{\min}} \right) 100 \quad (8.13)$$

Note that in Table 8.1 E_1 never exceeds ± 3 percent.

Representative contour plots of dimensionless pressure are shown for two quite different values of the ellipticity parameter k (8 and 1.25) in Figure 8.1. In these and all subsequent contour plots to be shown in this text, the $+$ symbol indicates

the center of the Hertzian contact zone. Note that the dimensionless representation of the X and Y coordinates adopted in the present work caused the actual Hertzian contact ellipse to become a circle regardless of the value of the ellipticity parameter. The Hertzian contact circle is shown in each figure by asterisks. On each figure is a key showing the contour labels and each corresponding value of dimensionless pressure. The inlet region is to the left and the exit region is to the right in each figure.

For $k = 8$ the maximum pressure is near the center of the contact; and even though the conditions are in the elastic region, there is no evidence of the pressure spike that is a feature of many theoretical solutions to the elastohydrodynamic problem. The pressure gradient at the exit end of the conjunction is much larger than that in the inlet region. For $k = 1.2$ a pressure spike is visible at the exit end of the contact.

Two contour plots of film thickness for the same values of ellipticity parameter, 8 and 1.25, are shown in Figure 8.2. For $k = 8$, Figure 8.2(a), the minimum film thickness is located directly behind the center of the Hertzian contact zone on the minor axis of the contact ellipse. For $k = 1.25$, Figure 8.2(b), two minimum-film-thickness regions occur in well-defined side lobes that follow, and are close to, the edge of the Hertzian contact ellipse. These results, showing the two side

lobes in which minimum-film-thickness areas occur, reproduce all the essential features of previously reported experimental observations based on optical interferometry (Cameron and Gohar, 1966).

The variations of pressure and film thickness in the direction of motion close to the midplane of the contact are shown in Figure 8.3 for three values of the ellipticity parameter. It is evident from Figure 8.3(a) that no pressure spike is predicted for $k = 6$, but this well-known feature of the theoretical solution to the EHL problem is evident for $k = 2.5$ and $k = 1.25$.

An interesting feature of the film thickness profiles shown in Figure 8.3(b) is the reentrant form of the film shape for $k = 1.25$. This can probably be attributed to lubricant compressibility, and similar profiles were reported for line contacts by Dowson and Higginson (1966). The film thickness in the central regions of elastohydrodynamic contacts lubricated by incompressible fluids is essentially constant; but when compressibility is considered, the local film thickness is reduced by an amount corresponding to the change in fluid volume with pressure.

8.3 Influence of Speed

If the surface velocity in the x direction u is changed, the dimensionless speed parameter U is modified as shown in equation (8.5), but the other dimensionless parameters

(k , W , G , and H) remain constant. The values at which these dimensionless parameters were held constant in the calculations performed to determine the influence of speed on film thickness were

$$k = 6, W = 0.7371 \times 10^{-6}, G = 4522 \quad (8.14)$$

Values of the dimensionless speed parameter U and the corresponding minimum film thickness H_{\min} as obtained from the EHL elliptical-contact theory developed in Chapter 7 are presented in Table 8.2. Calculations were performed for 15 values of the dimensionless speed parameter covering nearly two orders of magnitude. The solutions enabled the relationship between minimum film thickness and the speed parameter to be written in the form

$$\tilde{H}_{\min} = C_3 U^{C_4} \quad (8.15)$$

By applying a least-squares power fit to the 15 pairs of data $[(U_i, H_{\min,i}), i = 1, \dots, 15]$, the values of C_3 and C_4 were found to be $C_3 = 560$ and $C_4 = 0.6754 \approx 0.68$. The coefficient of determination ϵ for these results was excellent at 0.9998. Substituting the values of C_3 and C_4 into equation (8.15) gave the values of \tilde{H}_{\min} shown in Table 8.2. The percentage difference E_1 between the minimum film thickness obtained from the EHL elliptical-contact theory H_{\min} and the minimum film thickness obtained from the least-squares fit \tilde{H}_{\min} was determined

from equation (8.15), and the results are given in Table 8.2. Note that the variation of E_1 is less than ± 2 percent.

From the value obtained for C_4 and equation (8.15) the effect of dimensionless speed on dimensionless minimum film thickness can be written as

$$\tilde{H}_{\min} \propto U^{0.68} \quad (8.16)$$

The distributions of dimensionless pressure for two widely different values of the dimensionless speed parameter, 0.8416×10^{-12} and 0.5050×10^{-10} , are shown in Figure 8.4. For the lower speed of $U = 0.8416 \times 10^{-12}$ a pressure spike is evident at the exit end of the contact as shown in Figure 8.4(a). But for the higher speed of $U = 0.5050 \times 10^{-10}$ Figure 8.4(b) shows that no pressure spike emerges from the calculations. Another interesting feature of Figure 8.4 is that the pressure in the inlet region is higher at high speeds than at low speeds.

The dimensionless film thickness contours for values of U of 0.8416×10^{-12} and 0.5050×10^{-10} are shown in Figure 8.5. For the lower speed of $U = 0.8416 \times 10^{-12}$ the minimum film thickness is located along a crescent-shaped region close to the edge of the Hertzian contact ellipse as shown in Figure 8.5(a). For the higher speed of $U = 0.5050 \times 10^{-10}$ the minimum-film-thickness area appears in Figure 8.5(b) to be within the contact zone between the center and edge of the Hertzian contact ellipse.

The variation of pressure and film thickness in the direction of rolling quite close to the X axis near the midplane of the conjunction is shown in Figure 8.6 for three values of U . Recall that the values of the dimensionless load, materials, and ellipticity parameters were held constant at the levels noted in equation (8.14) for the calculations in this section. In Figure 8.6(a) the dashed line corresponds to the Hertzian pressure distribution. This figure shows that the pressure at any location in the inlet region rises as the speed increases, a result that is also consistent with the elastohydrodynamic theory for line or rectangular contacts. Furthermore, as the speed decreases, the height of the pressure spike decreases and the hydrodynamic pressures gradually approach the semielliptical form of the Hertzian contact stresses. Note that the location of the pressure spike moves downstream toward the edge of the Hertzian contact ellipse as the speed decreases. Similar results to those shown in Figure 8.6(a) have been presented for nominal line or rectangular contacts by Dowson and Higginson (1966).

A typical elastohydrodynamic film shape with an essentially parallel section in the central region is shown in Figure 8.6(b). There is little sign of a reentrant region in this case, except perhaps at the lowest speed. There is also a considerable change in the film thickness as the dimensionless speed is changed, as indicated by equation (8.16). This

illustrates most clearly the dominant effect of the dimensionless speed parameter U on the minimum film thickness in elastohydrodynamic contacts.

8.4 Influence of Load

Changes in the dimensionless load parameter W can be achieved while keeping the other parameters constant by changing only the normal applied load F in equation (8.3). The values at which the remaining parameters k , U , and G were held constant during the calculations were

$$k = 6, U = 0.1683 \times 10^{-11}, G = 4522 \quad (8.17)$$

Eight values of the dimensionless load parameter W covering just over one order of magnitude and the corresponding values of minimum film thickness H_{\min} as obtained from the EHL elliptical-contact theory developed in Chapter 7 are shown in Table 8.3. These eight pairs of data were used to determine an empirical relationship between the dimensionless load and the minimum film thickness, in the form

$$\tilde{H}_{\min} = C_5 W^{C_6} \quad (8.18)$$

By applying a least-squares power fit to the eight pairs of data $[(W_i, H_{\min,i}), i = 1, \dots, 8]$, the values of C_5 and C_6 were found to be $C_5 = 2.16 \times 10^{-6}$ and $C_6 = -0.0729 \approx -0.073$. The

coefficient of determination r^2 for these results was 0.9260 and was deemed to be quite good, but it was nevertheless the lowest value obtained in deriving the minimum-film-thickness equation (8.23). Substituting the values for C_5 and C_6 into equation (8.18) gave the values of \tilde{H}_{\min} shown in Table 8.3. The percentage difference E_1 between the minimum film thickness obtained from the EHL elliptical-contact theory H_{\min} and the minimum film thickness from the least-squares-fit equation \tilde{H}_{\min} was ascertained by the application of equation (8.13), and the results are given in Table 8.3. The values of E_1 never exceed ± 3 percent.

From the value of C_6 and equation (8.18) the effect of load on minimum film thickness can be written as

$$\tilde{H}_{\min} \propto W^{-0.073} \quad (8.19)$$

This very small negative exponent is again consistent with the findings of earlier theoretical and experimental studies of elastohydrodynamic lubrication. It shows that an increase in load will produce a decrease in minimum film thickness but that the decrease is remarkably small. The elastohydrodynamic film, once formed, is thus quite insensitive to changes in load if all other factors remain constant.

The distributions of dimensionless pressure for the two extreme values of dimensionless load parameter considered ($W = 0.1106 \times 10^{-6}$ and $W = 0.1290 \times 10^{-5}$) are shown in contour

form in Figure 8.7. At the lower load of $W = 0.1106 \times 10^{-6}$ (Figure 8.7(a)) the conditions do not promote the formation of a pressure spike, but this distinctive feature of many elastohydrodynamic solutions is evident at the higher load of $W = 0.1290 \times 10^{-5}$ (Figure 8.7(b)).

Contour plots of dimensionless film thickness for the same two values of the dimensionless load parameter considered in deriving the pressure distributions depicted in Figure 8.7 are shown in Figure 8.8. For the lower load of $W = 0.1106 \times 10^{-6}$ the minimum film thickness shown in Figure 8.8(a) occurs directly behind the center of the contact. For the higher load of $W = 0.1290 \times 10^{-5}$ the minimum film thickness is found in regions broadly represented by a crescent located near the edge of the Hertzian contact ellipse, as shown in Figure 8.8(b).

The variation of pressure and film thickness in the direction of motion along a line close to the midplane of the conjunction is shown in Figure 8.9 for three values of dimensionless load parameter. The values of the dimensionless speed, materials, and ellipticity parameters were held fixed as described by equation (8.17) for all computations at various loads. Note from Figure 8.9(a) that the pressure at any location in the inlet region falls as the load increases. For the highest load of $W = 1.106 \times 10^{-6}$ the film thickness depicted in Figure 8.9(b) rises between the central region and the outlet restriction in the same manner as shown previously

in Figure 8.3(b). Again this reentrant effect is attributed to lubricant compressibility. Note also that at $W = 0.5528 \times 10^{-6}$ the film thickness is slightly smaller than at $W = 1.106 \times 10^{-6}$. This somewhat curious result is linked to the fact that the location of the minimum film thickness also changes drastically over this load range. At the lower load the minimum film thickness is located on the midplane of the conjunction downstream from the center of the contact; at the higher load it moves to the side lobes as described earlier.

8.5 Influence of Material Properties

A study of the influence of the dimensionless materials parameter G on minimum film thickness has to be approached with caution since in practice it is not possible to change the physical properties of the materials, and hence the value of G , without influencing the other dimensionless parameters considered earlier. Equations (8.3), (8.5), and (8.7) show that as either the materials of the solids (as expressed in E') or the lubricants (as expressed in η_0 and $p_{iv,as}$) are varied, not only does the materials parameter G change, but so do the dimensionless speed U and load W parameters. Only the ellipticity parameter can be held fixed, and for all results presented in this section $k = 6$ was adopted.

The results obtained from calculations performed for four values of the dimensionless materials parameter are summarized in Table 8.4. The general form of these results, showing how the minimum film thickness is a function of the dimensionless materials parameter, is written as

$$\tilde{C} = C_7 G^{C_8} \quad (8.20)$$

where

$$\tilde{C} = \frac{H_{\min}}{(1 - e^{-0.68k})U^{0.68}W^{-0.073}} \quad (8.21)$$

In equation (8.21) the exponents have been rounded off to two significant figures so that any error could be absorbed in C_7 , given in equation (8.20). By applying a least-squares power fit to the four pairs of data, the values of C_7 and C_8 were found to be $C_7 = 3.69$ and $C_8 = 0.487 \approx 0.49$. The coefficient of determination for these results was 0.9980, which is excellent. Substituting values of C_7 and C_8 into equation (8.20) gives the values of \tilde{H}_{\min} shown in Table 8.4. The percentage difference E_1 between \tilde{H}_{\min} and H_{\min} shown in Table 8.4 varies by only 2 percent at all times. Therefore the effect of the dimensionless materials parameter on the dimensionless film thickness can be written with adequate accuracy as

$$\tilde{H}_{\min} \propto G^{0.49} \quad (8.22)$$

8.6 Minimum-Film-Thickness Formula

The proportionality expression equations (8.12), (8.16), (8.19), and (8.22) have established how the minimum film thickness varies with the ellipticity, speed, load, and materials parameters, respectively. This enables a composite minimum-film-thickness formula for a fully flooded, isothermal elastohydrodynamic elliptical contact to be written as

$$\tilde{H}_{\min} = 3.63 U^{0.68} G^{0.49} W^{-0.073} (1 - e^{-0.68k}) \quad (8.23)$$

In equation (8.23) the constant 3.63 is different from $C_7 = 3.69$ mentioned earlier to account for the rounding off of the materials-parameter exponent. It is sometimes more convenient to express the side-leakage factor in equation (8.23) in terms of the radius-of-curvature ratio R_y/R_x instead of the ellipticity parameter. This can be done by using equation (3.28), or

$$k = 1.03 \left(\frac{R_y}{R_x} \right)^{0.64} \quad (3.28)$$

where

$$\frac{1}{R_y} = \frac{1}{r_{ay}} + \frac{1}{r_{by}} \quad (8.24)$$

$$\frac{1}{R_x} = \frac{1}{r_{ax}} + \frac{1}{r_{bx}} \quad (8.25)$$

There is considerable merit in using equation (3.28) since it avoids the need to evaluate elliptic integrals of the first and second kinds in determining k . The minimum film thickness can thus be derived directly from a knowledge of the radii of curvature of the contacting bodies (r_{ax} , r_{ay} , r_{bx} , and r_{by}).

It is interesting to compare the Hamrock and Dowson (1977a) elliptical-contact, minimum-film-thickness formula equation (8.23) with the corresponding equation generated by Dowson (1968) for nominal line or rectangular contacts.

$$H_{\min,r} = 2.65 U^{0.70} G^{0.54} W^{-0.13} \quad (8.26)$$

The powers of U , G , and W in equations (8.23) and (8.26) are quite similar considering the different numerical procedures on which they are based. It is also worth noting that the power of W in equation (8.23) is extremely close to the value of -0.074 produced by Archard and Cowking (1966) in their studies of elliptical contacts.

8.7 Application of the Elastohydrodynamic Minimum-Film-Thickness Formula for Line Contacts to Elliptical-Contact Situations

The dimensionless minimum-film-thickness formula for line or rectangular contacts can, under certain circumstances, be used to estimate the film thickness in elliptical contacts with acceptable accuracy. Indeed, equations like equation (8.26)

were used for this purpose before elliptical-contact film thickness equations presented in this text became available.

The problem is to determine the appropriate value of the load per unit length of the equivalent cylinder W_r to be used in equation (8.26). The expression for W_r employed in the line-contact film thickness formula is

$$W_r = \frac{F^*}{E' R_x} \quad (8.27)$$

where F^* is the load per unit length of the cylinder, or

$$F^* = \frac{F}{l} \quad (8.28)$$

The value of the applied load F on an elastohydrodynamic conjunction is usually known, but it is important that the basis for selecting the effective length of the contact should be specified.

In a dry elliptical contact the pressure along the principal axis varies according to the Hertzian formula for a semi-ellipse, whereas in a line contact the pressure remains constant in the axial direction. This at once suggests two approaches to the determination of an effective length of the elliptical contact; one that ensures that the maximum pressures in the elliptical and rectangular contacts are the same, and the other that yields the same mean pressure in the two situations.

8.7.1 Determination of the Effective Length of a Rectangular Contact Required to Generate the Same Maximum Pressure as That Produced in an Elliptical Contact

The maximum Hertzian contact pressure in an elliptical contact can be written as (Chapter 3),

$$P_{\max} = \frac{3F}{2\pi ab} \quad (3.6)$$

where

$$b = \left(\frac{6\phi FR}{\pi k E'} \right)^{1/3} \quad (3.14)$$

Likewise, the maximum Hertzian pressure in a rectangular contact (Harris, 1966) is given by

$$P_{\max, r} = \frac{2F}{\pi \ell b_r} \quad (8.29)$$

where

$$b_r = \left(\frac{8FR}{\pi \ell E'} \right)^{1/2} \quad (8.30)$$

Thus, if the maximum pressures are assumed to be equal in the two cases, equations (3.6) and (8.29) can be equated to yield

$$\ell = \frac{4a}{3} \left(\frac{b}{b_r} \right) \quad (8.31)$$

When the expressions for b and b_r are introduced from equations (3.14) and (8.30) this becomes

$$\ell = \frac{2}{9} (6)^{2/3} a^2 \left(\frac{\pi E'}{FR} \right)^{1/3} \left(\frac{\phi}{k} \right)^{2/3} \quad (8.32)$$

The semimajor axis a in a dry elliptical contact has been written in Chapter 3 as

$$a = \left(\frac{6k^2 \mathcal{E} FR}{\pi E'} \right)^{1/3} \quad (3.13)$$

Hence

$$\left(\frac{\pi E'}{FR} \right)^{1/3} = \frac{1}{a} (6k^2 \mathcal{E})^{1/3} \quad (8.33)$$

When equation (8.33) is introduced into equation (8.32), it is seen that the effective length of the cylinder required to yield the same maximum pressure as the actual elliptical contact is given by

$$\ell = \frac{4a}{3} \mathcal{E} = 1.33 a \mathcal{E} \quad (8.34)$$

For long, thin contact ellipses in which $R_y \gg R_x$ the magnitudes of b and b_r do not differ greatly and \mathcal{E} approaches unity, as shown in Figure 3.3. In this case ℓ approaches $\left(\frac{4}{3} a \right)$, a result that can be obtained directly from equation (8.31). If the approximation for \mathcal{E} from equation (3.19) is introduced, equation (8.34) can be written as

$$\ell \approx \frac{4}{3} a \left(1.0003 + \frac{0.5968}{R_y/R_x} \right) \quad (8.35)$$

8.7.2 Determination of Effective Length of a Rectangular Contact Required to Generate the Same Mean Contact Pressure as That Produced in an Elliptical Contact

The mean Hertzian contact pressure in an elliptical contact is given by

$$P_m = \frac{F}{\pi ab} \quad (8.36)$$

where b is given by equation (3.14) as before.

Similarly the mean Hertzian contact pressure in a rectangular contact of length ℓ is given by

$$P_{m,r} = \frac{F}{2\ell b_r} \quad (8.37)$$

where b_r is represented by equation (8.30).

If the mean pressures are assumed to be equal in the two cases, equations (8.36) and (8.37) can be equated to give

$$\ell = \frac{\pi a}{2} \left(\frac{b}{b_r} \right) \quad (8.38)$$

When equations (3.14) and (8.30) are introduced for b and b_r , this becomes

$$\ell = \frac{\pi^2}{32} (6)^{2/3} a^2 \left(\frac{\pi E'}{FR} \right)^{1/3} \left(\frac{\epsilon}{k} \right)^{2/3} \quad (8.39)$$

Once again equation (8.33) can be introduced to replace the first term in brackets in equation (8.39) and thus to yield

$$\ell = \frac{3}{16} \pi^2 a \epsilon^2 \quad (8.40)$$

where

$$\mathcal{E} = 1.0003 + \frac{0.5968}{R_y/R_x} \quad (3.29)$$

Equation (8.40) should be used for the effective length of a cylinder required to yield the same mean pressure as that encountered in the real elliptical contact. For large values of R_y/R_x a long, thin contact ellipse is found and \mathcal{E} approaches unity. In this case ℓ approaches $1.85 a$.

For these long, thin elliptical contacts the length of the cylindrical contacts required to give comparable maximum and mean pressures to those in the real contact approach $1.33 a$ and $1.85 a$, respectively.

8.8 Central-Film-Thickness Formula

In practical situations there is considerable interest in the central as well as the minimum film thickness in elastohydrodynamic contacts. This is particularly true when traction is considered since the surfaces in relative motion are separated by a film of almost constant thickness that is well represented by the central value over much of the Hertzian contact zone. The procedure used to obtain an expression for the central film thickness was the same as that used in obtaining the minimum film thickness and will not be repeated here. The central-film-thickness formula obtained from the numerical results is

$$\tilde{H}_c = 2.69 U^{0.67} G^{0.53} W^{-0.067} (1 - 0.61e^{-0.73k}) \quad (8.41)$$

Note that the exponent on the dimensionless load parameter is still negative in this equation but that the numerical value is slightly smaller than that determined for the minimum-film-thickness expression. This contrasts with the numerical study of Ranger, et al. (1975), who found a small but positive exponent on W in their formula for the central film thickness.

A similar expression for nominal line or rectangular conjunctions based on a large number of solutions to the elastohydrodynamic problem for cylinders has been presented by Dowson and Toyoda (1979) and can be written as

$$H_c = 3.06 U^{0.69} G^{0.56} W^{-0.10} \quad (8.42)$$

The agreement between the predictions of equation (8.41) with $k = \infty$ and those of equation (8.42) is remarkably good.

8.9 Elastohydrodynamic Lubrication of the Ball-Race Contact in a Ball Bearing

High-speed, thrust-loaded, angular-contact ball bearings require sufficient loading to prevent gross sliding motion (i.e., skidding between the ball and the inner race). As discussed in Chapter 3 (Section 3.5.2) such sliding can lead to surface distress and eventually to bearing destruction. The degree of distress depends on the adequacy of ball-race

lubrication, as determined by the elastohydrodynamic minimum lubricant film thickness relative to the roughness of the sliding surfaces. Harris (1971) appears to have been the first to consider the effects of elastohydrodynamic lubrication in determining friction forces between balls and races and to predict the threshold thrust loading at which skidding might occur. The approach used by Harris (1971) is adopted here in association with the elastohydrodynamic lubrication results given elsewhere in this chapter to facilitate an analysis of the lubrication of ball-race contacts in ball bearings. The assumptions of isothermal, Newtonian behavior of the lubricant in the ball-race contacts used elsewhere in this chapter are also used here.

The mathematical model used is based on the forces and moments shown in Figure 8.10. The coordinate system is shown in Figure 8.11. Note that the ball motion at any position is described by the angular speeds $\omega_{x'}$, $\omega_{y'}$, and $\omega_{z'}$ about three mutually orthogonal axes through the ball center such that

$$\omega_{x'} = \omega_B \cos \alpha^* \cos \beta^* \quad (8.43)$$

$$\omega_{y'} = \omega_B \cos \alpha^* \sin \beta^* \quad (8.44)$$

$$\omega_{z'} = \omega_B \sin \alpha^* \quad (8.45)$$

In these equations α^* is the pitch angle of the ball speed vector relative to the x' axis, and β^* defines the yaw angle, also called the skew angle.

The surface shear or frictional stresses occurring in the ball-race contacts can be defined in terms of the sliding velocities and the lubricant pressure and film thickness acting within these contacts. Since sliding velocities, pressures, and film thickness vary from point to point within the contact, the tangential force in any direction acting at each contact must be determined by integrating the shear stresses over the elliptical contact surface. The sliding velocities in the y' direction are given by

$$v_{ym} = \frac{d}{2} \left[\frac{\omega_m d'_e}{d} + (c_m \omega_m - \omega_{x'}) \cos \beta_m - \omega_{z'} \sin \beta_m \right] \quad (8.46)$$

where $m = 0$ or i and $c_0 = 1$ and $c_i = -1$. Also the sliding velocity in the direction of the major axis owing to the rotation about the y' axis is

$$v_{xo} = v_{xi} - \frac{d}{2} \omega_{y'} \quad (8.47)$$

In equation (8.46) the pitch diameter d'_e for the situation resulting after the forces have been applied was defined in Chapter 3 (equation (3.107)). Furthermore the contact angles β_0 and β_i are defined by equations (3.96)

to (3.99), with the exception that in this section only thrust loads are considered, so that $\delta_r = 0$.

If it is assumed that the sliding velocity varies linearly with the distance between the surfaces, the Newtonian concept of viscous fluid friction reduces to

$$\tau = \eta \frac{v}{h} \quad (8.48)$$

The viscosity is a function of pressure as described by equation (5.50). Therefore the viscosity and film thickness will vary within the contact, and the values to be used are those obtained from the elastohydrodynamic analysis. The tangential forces are defined as

$$\tilde{T}_{ym} = a_m b_m \int_{-1}^1 \int_{-\sqrt{1-X^2}}^{\sqrt{1-X^2}} \tau_{ym} dY dX \quad (8.49)$$

$$\tilde{T}_{xm} = a_m b_m \int_{-1}^1 \int_{-\sqrt{1-Y^2}}^{\sqrt{1-Y^2}} \tau_{xm} dX dY \quad (8.50)$$

Because of its orbital speed each ball must overcome a viscous drag force imposed by the lubricant within the bearing cavity. It can be assumed that the drag caused by the gaseous atmosphere is insignificant; however, the lubricant viscous drag depends on the quantity of the lubricant dispersed in the bearing cavity. Hence the effective fluid within the cavity is a

gas-oil mixture that has an effective viscosity and an effective specific gravity. The viscous drag force acting on an orbiting ball can be written as

$$\tilde{T}_v = \frac{\rho A_v C_v (d'_e \omega_c)^2}{8g} \quad (8.51)$$

where

A_v = drag area of ball

C_v = drag coefficient

g = gravitational constant

From Figure 8.10 the following conditions of force equilibrium must be satisfied for steady-state operation of the bearing:

$$F_o \cos \beta_o - F_i \cos \beta_i - \tilde{T}_{xo} \sin \beta_o + \tilde{T}_{xi} \sin \beta_i - F_c = 0 \quad (8.52)$$

$$F_o \sin \beta_o - F_i \sin \beta_i + \tilde{T}_{xo} \cos \beta_o - \tilde{T}_{xi} \cos \beta_i = 0 \quad (8.53)$$

$$\tilde{T}_{yo} - \tilde{T}_{yi} + \tilde{T}_v = 0 \quad (8.54)$$

In equation (8.52) F_c is the centrifugal force acting on the ball and is defined by equation (3.106). Cage-to-ring and cage-to-ball forces have not been considered.

The moment equilibrium conditions to be satisfied are

$$\frac{d}{2} \int_{-1}^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} (a_o b_o \tau_{xo} + a_i b_i \tau_{xi}) dx dy - M_{gy} = 0 \quad (8.55)$$

$$\frac{d}{2} \int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (a_o b_o \tau_{yo} \cos \beta_o + a_i b_i \cos \beta_i \tau_{yi}) dY dX = 0 \quad (8.56)$$

$$\frac{d}{2} \int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (a_o b_o \tau_{yo} \sin \beta_o + a_i b_i \tau_{yi} \sin \beta_i) dY dX$$

$$- M_{gz'} = 0 \quad (8.57)$$

where

$$M_{gy'} = \bar{J} \omega_c \omega_{z'}$$

$$M_{gz'} = \bar{J} \omega_c \omega_{y'}$$

and \bar{J} is the polar moment of inertia. As a final condition of equilibrium, it is recognized that for thrust-loaded bearings

$$F_i \sin \beta_i + \tilde{T}_{xi} \cos \beta_i - \frac{F_t}{n} = 0 \quad (8.58)$$

Figure 8.12 indicates the relative location of the ball center and the race groove curvature centers for a combined radial and thrust-loaded bearing operating at high speeds. If the radial displacement δ_r is set equal to zero and the results from this change are made to Figure 8.12, the position of the ball for a thrust-loaded bearing can be determined. From this figure the following relationships can be developed:

$$L_4^2 + L_3^2 - \left[d(f_o - 0.5) + \delta_o \right]^2 = 0 \quad (8.59)$$

$$\begin{aligned} & \left(D \cos \beta_f - L_4 \right)^2 + \left(D \sin \beta_f + \delta_t - L_3 \right)^2 \\ & - \left[(f_i - 0.5)d + \delta_i \right]^2 = 0 \quad (8.60) \end{aligned}$$

Equations (8.52) to (8.60) constitute a set of nine simultaneous, nonlinear equations that can be solved by numerical methods for the variables L_3 , L_4 , δ_i , δ_o , δ_t , α^* , β^* , ω_B , and ω_C .

The test data of Shevchenko and Bolin (1957) and the analytical results of Harris (1971) are compared in Figure 8.12. Analytical results according to the race-control hypothesis discussed in Chapter 3 are also indicated. Clearly the Harris (1971) analysis, which uses tangential forces predicted by an elastohydrodynamic-lubrication analysis much like that discussed in this section, approximates the test data more closely than the race-control analytical method.

The key elements in predicting bearing performance more accurately are, as mentioned by Anderson (1978), better predictions of film thickness and tractive force. The elastohydrodynamic lubrication film thickness equations for elliptical contacts given in this chapter are thus offered to support a step toward the better prediction of bearing performance. Furthermore the study of elastohydrodynamic lubrication has stimulated

research on lubricant rheology that is leading to the development of better models for calculating tractive forces. Considerable progress has thus been made in evaluating the overall performance of ball bearings since Harris pioneered the way in 1971. A revised version of Harris analysis was used in developing a computer program called SHABERTH, which incorporates actual traction data from a disk machine. This computer program is used to correlate predicted and measured bearing operating temperatures and power loss. Since its beginning in the mid-1970's SHABERTH has been updated through a number of versions.

An excellent discussion of the development of computer programs for the analysis of ball bearing performance has been given by Pirvics (1980). Pirvics points out that before the early 1960's there were no rolling-element computer programs of note. For a number of years thereafter and into the mid-1970's efforts were made to understand and simulate particular bearing types as well as the nature of elastohydrodynamic lubrication. During this period (1965-1975) the major concern was to improve understanding of the basic behavior of individual contacts within a bearing and the way in which they affect the performance of a complete bearing. This led to a system analysis that could be integrated into the surrounding machinery and that resulted in computer programs capable of treating the time-transient thermomechanics of load-carrying assemblies. Therefore by the end of the 1970's the significance and practical

necessity of the system design approach for detailing the time-transient, dynamic effects were recognized.

8.10 Closure

By using the procedures outlined in Chapter 7 the influence of the ellipticity parameter and the dimensionless speed, load, and materials parameters on minimum film thickness has been investigated. The ellipticity parameter k was varied from 1 (a ball-on-plane configuration) to 8 (a configuration approaching a rectangular contact). The dimensionless speed parameter was varied over a range of nearly two orders of magnitude, and the dimensionless load parameter over a range of one order of magnitude. Situations equivalent to using solid materials of bronze, steel, and silicon nitride and lubricants of paraffinic and naphthenic mineral oils were considered in an investigation of the role of the dimensionless materials parameter. Thirty-four cases were used to generate the following minimum- and central-film-thickness relationships:

$$\tilde{H}_{\min} = 3.63 U^{0.68} G^{0.49} W^{-0.073} (1 - e^{-0.68k})$$

$$\tilde{H}_c = 2.69 U^{0.67} G^{0.53} W^{-0.067} (1 - 0.61e^{-0.73k})$$

Contour plots have been presented that indicate in detail the pressure distribution and the film shape. In some solutions

the characteristic pressure spikes generated in earlier solutions to the line-contact elastohydrodynamic problem were in evidence. The theoretical solutions for the film thickness have all the essential features of previously reported experimental observations based on optical interferometry.

The results presented in this chapter have permitted the generation of a satisfactory theoretical film thickness equation for elastohydrodynamic elliptic contacts operating under fully flooded conditions. The exponents on the various dimensionless parameters governing the minimum film thickness in such conjunctions are quite similar to those developed by Dowson (1968) for nominal line or rectangular contacts. The most dominant exponent occurs on the speed parameter, but the exponent on the load parameter is very small and negative. The materials parameter also carried a significant exponent, although the range of this variable in engineering situations is limited.

Expressions have been developed for the selection of equivalent lengths of line or rectangular contacts that permit elastohydrodynamic film thickness equations for line contacts to be applied to elliptical contacts in some situations. The expressions developed enable comparisons to be made between line or rectangular and elliptical-contact situations on the basis of equal maximum or mean contact pressures.

Perhaps the most significant feature of the proposed minimum-film-thickness formula is that it can be applied to any contacting solids that present an elliptical Hertzian contact re-

gion. Many machine elements – particularly rolling-element bearings – possess such geometry, and it is expected that the new minimum-film-thickness equation will find ready application in such fields.

Finally a brief outline of the application of elastohydrodynamic theory to the ball-race contact in a ball bearing and a discussion of the influence of this theory on bearing mechanics are presented to illustrate the usefulness of the elliptical-contact solutions developed in this chapter.

SYMBOLS

A	constant used in equation (3.113)
$A^*, B^*, C^*,$ D^*, L^*, M^*	relaxation coefficients
A_v	drag area of ball, m^2
a	semimajor axis of contact ellipse, m
\bar{a}	$a/2\bar{m}$
B	total conformity of bearing
b	semiminor axis of contact ellipse, m
\bar{b}	$b/2\bar{m}$
C	dynamic load capacity, N
C_v	drag coefficient
C_1, \dots, C_8	constants
c	19,609 N/cm ² (28,440 lbf/in ²)
\bar{c}	number of equal divisions of semimajor axis
D	distance between race curvature centers, m
\tilde{D}	material factor
\bar{D}	defined by equation (5.63)
De	Deborah number
d	ball diameter, m
\bar{d}	number of divisions in semiminor axis
d_a	overall diameter of bearing (Figure 2.13), m
d_b	bore diameter, m
d_e	pitch diameter, m
d'_e	pitch diameter after dynamic effects have acted on ball, m
d_i	inner-race diameter, m
d_o	outer-race diameter, m

E	modulus of elasticity, N/m^2
E'	effective elastic modulus, $2 / \left(\frac{1 - \nu_a^2}{E_a} + \frac{1 - \nu_b^2}{E_b} \right)$, N/m^2
E_a	internal energy, m^2/s^2
\tilde{E}	processing factor
E_1	$[(\tilde{H}_{min} - H_{min})/H_{min}] \times 100$
\mathcal{E}	elliptic integral of second kind with modulus $(1 - 1/k^2)^{1/2}$
$\overline{\mathcal{E}}$	approximate elliptic integral of second kind
e	dispersion exponent
F	normal applied load, N
F^*	normal applied load per unit length, N/m
\tilde{F}	lubrication factor
\overline{F}	integrated normal applied load, N
F_c	centrifugal force, N
F_{max}	maximum normal applied load (at $\psi = 0$), N
F_r	applied radial load, N
F_t	applied thrust load, N
F_ψ	normal applied load at angle ψ , N
\mathcal{F}	elliptic integral of first kind with modulus $(1 - 1/k^2)^{1/2}$
$\overline{\mathcal{F}}$	approximate elliptic integral of first kind
f	race conformity ratio
f_b	rms surface finish of ball, m
f_r	rms surface finish of race, m
G	dimensionless materials parameter, αE
G^*	fluid shear modulus, N/m^2
\tilde{G}	hardness factor
g	gravitational constant, m/s^2

g_E	dimensionless elasticity parameter, $W^{8/3}/U^2$
g_V	dimensionless viscosity parameter, GW^3/U^2
H	dimensionless film thickness, h/R_x
\hat{H}	dimensionless film thickness, $H(W/U)^2 = F^2 h/u^2 \eta_0^2 R_x^3$
H_c	dimensionless central film thickness, h_c/R_x
$H_{c,s}$	dimensionless central film thickness for starved lubrication condition
H_f	frictional heat, N m/s
H_{min}	dimensionless minimum film thickness obtained from EHL elliptical-contact theory
$H_{min,r}$	dimensionless minimum film thickness for a rectangular contact
$H_{min,s}$	dimensionless minimum film thickness for starved lubrication condition
\tilde{H}_c	dimensionless central film thickness obtained from least-squares fit of data
\tilde{H}_{min}	dimensionless minimum film thickness obtained from least-squares fit of data
\bar{H}_c	dimensionless central-film-thickness - speed parameter, $H_c U^{-0.5}$
\bar{H}_{min}	dimensionless minimum-film-thickness - speed parameter, $H_{min} U^{-0.5}$
\bar{H}_0	new estimate of constant in film thickness equation
h	film thickness, m
h_c	central film thickness, m
h_i	inlet film thickness, m

h_m	film thickness at point of maximum pressure, where $dp/dx = 0$, m
h_{min}	minimum film thickness, m
h_0	constant, m
I_d	diametral interference, m
I_p	ball mass moment of inertia, $m N s^2$
I_r	integral defined by equation (3.76)
I_t	integral defined by equation (3.75)
J	function of k defined by equation (3.8)
J^*	mechanical equivalent of heat
\bar{J}	polar moment of inertia, $m N s^2$
K	load-deflection constant
k	ellipticity parameter, a/b
\bar{k}	approximate ellipticity parameter
\tilde{k}	thermal conductivity, $N/s \text{ } ^\circ C$
k_f	lubricant thermal conductivity, $N/s \text{ } ^\circ C$
L	fatigue life
L_a	adjusted fatigue life
L_t	reduced hydrodynamic lift, from equation (6.21)
L_1, \dots, L_4	lengths defined in Figure 3.11, m
L_{10}	fatigue life where 90 percent of bearing population will endure
L_{50}	fatigue life where 50 percent of bearing population will endure
ℓ	bearing length, m
$\bar{\ell}$	constant used to determine width of side-leakage region
M	moment, Nm

M_g	gyroscopic moment, Nm
M_p	dimensionless load-speed parameter, $WU^{-0.75}$
M_s	torque required to produce spin, N m
m	mass of ball, $N s^2/m$
m^*	dimensionless inlet distance at boundary between fully flooded and starved conditions
\tilde{m}	dimensionless inlet distance (Figures 7.1 and 9.1)
\bar{m}	number of divisions of semimajor or semiminor axis
m_W	dimensionless inlet distance boundary as obtained from Wedeven, et al. (1971)
N	rotational speed, rpm
n	number of balls
n^*	refractive index
\bar{n}	constant used to determine length of outlet region
P	dimensionless pressure
P_D	dimensionless pressure difference
P_d	diametral clearance, m
P_e	free endplay, m
P_{Hz}	dimensionless Hertzian pressure, N/m^2
p	pressure, N/m^2
P_{max}	maximum pressure within contact, $3F/2\pi ab$, N/m^2
$P_{iv,as}$	isoviscous asymptotic pressure, N/m^2
Q	solution to homogeneous Reynolds equation
Q_m	thermal loading parameter
\bar{Q}	dimensionless mass flow rate per unit width, $q\eta_0/\rho_0 E' R^2$
q_f	reduced pressure parameter
q_x	volume flow rate per unit width in x direction, m^2/s

q_y	volume flow rate per unit width in y direction, m^2/s
R	curvature sum, m
R_a	arithmetical mean deviation defined in equation (4.1), m
R_c	operational hardness of bearing material
R_x	effective radius in x direction, m
R_y	effective radius in y direction, m
r	race curvature radius, m
$\left. \begin{matrix} r_{ax}, r_{bx}, \\ r_{ay}, r_{by} \end{matrix} \right\}$	radii of curvature, m
r_c, ϕ_c, z	cylindrical polar coordinates
r_s, θ_s, ϕ_s	spherical polar coordinates
\bar{r}	defined in Figure 5.4
S	geometric separation, m
S^*	geometric separation for line contact, m
S_0	empirical constant
s	shoulder height, m
T	τ_0/p_{max}
\tilde{T}	tangential (traction) force, N
T_m	temperature, $^{\circ}C$
T_b^*	ball surface temperature, $^{\circ}C$
T_f^*	average lubricant temperature, $^{\circ}C$
ΔT^*	ball surface temperature rise, $^{\circ}C$
T_1	$(\tau_0/p_{max})_{k=1}$
T_v	viscous drag force, N
t	time, s
t_a	auxiliary parameter
u_B	velocity of ball-race contact, m/s

u_c	velocity of ball center, m/s
U	dimensionless speed parameter, $\eta_0 u / E' R_x$
u	surface velocity in direction of motion, $(u_a + u_b)/2$, m/s
\bar{u}	number of stress cycles per revolution
Δu	sliding velocity, $u_a - u_b$, m/s
v	surface velocity in transverse direction, m/s
W	dimensionless load parameter, $F/E'R^2$
w	surface velocity in direction of film, m/s
X	dimensionless coordinate, x/R_x
Y	dimensionless coordinate, y/R_x
X_t, Y_t	dimensionless grouping from equation (6.14)
X_a, Y_a, Z_a	external forces, N
Z	constant defined by equation (3.48)
Z_1	viscosity pressure index, a dimensionless constant
$\left. \begin{array}{l} x, \tilde{x}, \bar{x}, \bar{x}_1 \\ y, \tilde{y}, \bar{y}, \bar{y}_1 \\ z, \tilde{z}, \bar{z}, \bar{z}_1 \end{array} \right\}$	coordinate system
α	pressure-viscosity coefficient of lubrication, m^2/N
α_a	radius ratio, R_y/R_x
β	contact angle, rad
β_f	free or initial contact angle, rad
β'	iterated value of contact angle, rad
Γ	curvature difference
γ	viscous dissipation, $N/m^2 \cdot s$
$\dot{\gamma}$	total strain rate, s^{-1}
$\dot{\gamma}_e$	elastic strain rate, s^{-1}
$\dot{\gamma}_v$	viscous strain rate, s^{-1}

γ_a	flow angle, deg
δ	total elastic deformation, m
δ^*	lubricant viscosity temperature coefficient, $^{\circ}\text{C}^{-1}$
δ_D	elastic deformation due to pressure difference, m
δ_r	radial displacement, m
δ_t	axial displacement, m
δ_x	displacement at some location x , m
$\bar{\delta}$	approximate elastic deformation, m
$\tilde{\delta}$	elastic deformation of rectangular area, m
ϵ	coefficient of determination
ϵ_1	strain in axial direction
ϵ_2	strain in transverse direction
ζ	angle between ball rotational axis and bearing centerline (Figure 3.10)
ζ_a	probability of survival
η	absolute viscosity at gauge pressure, N s/m^2
$\bar{\eta}$	dimensionless viscosity, η/η_0
η_0	viscosity at atmospheric pressure, N s/m^2
η_{∞}	$6.31 \times 10^{-5} \text{ N s/m}^2 (0.0631 \text{ cP})$
θ	angle used to define shoulder height
Λ	film parameter (ratio of film thickness to composite surface roughness)
λ	equals 1 for outer-race control and 0 for inner-race control
λ_a	second coefficient of viscosity
λ_b	Archard-Cowking side-leakage factor, $(1 + 2/3 \alpha_a)^{-1}$
λ_c	relaxation factor

μ	coefficient of sliding friction
μ^*	$\overline{\rho}/\overline{\eta}$
ν	Poisson's ratio
ξ	divergence of velocity vector, $(\partial u/\partial x) + (\partial v/\partial y) + (\partial w/\partial z)$, s^{-1}
ρ	lubricant density, $N\ s^2/m^4$
$\overline{\rho}$	dimensionless density, ρ/ρ_0
ρ_0	density at atmospheric pressure, $N\ s^2/m^4$
σ	normal stress, N/m^2
σ_1	stress in axial direction, N/m^2
τ	shear stress, N/m^2
τ_0	maximum subsurface shear stress, N/m^2
$\tilde{\tau}$	shear stress, N/m^2
$\tilde{\tau}_e$	equivalent stress, N/m^2
$\tilde{\tau}_L$	limiting shear stress, N/m^2
Φ	ratio of depth of maximum shear stress to semiminor axis of contact ellipse
Φ^*	$\rho H^{3/2}$
Φ_1	$(\Phi)_{k=1}$
ϕ	auxiliary angle
ϕ_T	thermal reduction factor
ψ	angular location
ψ_ℓ	limiting value of ψ
Ω_i	absolute angular velocity of inner race, rad/s
Ω_o	absolute angular velocity of outer race, rad/s
ω	angular velocity, rad/s
ω_B	angular velocity of ball-race contact, rad/s
ω_b	angular velocity of ball about its own center, rad/s

ω_c angular velocity of ball around shaft center, rad/s

ω_s ball spin rotational velocity, rad/s

Subscripts:

a solid a

b solid b

c central

bc ball center

IE isoviscous-elastic regime

IR isoviscous-rigid regime

i inner race

K Kapitza

min minimum

n iteration

o outer race

PVE piezoviscous-elastic regime

PVR piezoviscous-rigid regime

r for rectangular area

s for starved conditions

x,y,z coordinate system

Superscript:

(\sim) approximate

REFERENCES

- Abbott, E. J. and Firestone, F. A. (1933) Specifying Surface Quality, Mech. Eng., 55, 569-572.
- Agricola, G. (1556) De Re Metallica, Basel.
- Aihara, S. and Dowson, D. (1979) "A Study of Film Thickness in Grease Lubricated Elastohydrodynamic Contacts," Proceedings of Fifth Leeds-Lyon Symposium on Tribology on 'Elastohydrodynamics and Related Topics', D. Dowson, C. M. Taylor, M. Godet, and D. Berthe, eds., Mechanical Engineering Publications, Ltd., 104-115.
- Allan, R. K. (1945) Rolling Bearings, Sir Isaac Pitman & Sons, London.
- Alsaad, M., Bair, S., Sanborn, D. M., and Winer, W. O. (1978) "Glass Transitions in Lubricants: Its Relation to Elastohydrodynamic Lubrication (EHD)," J. Lubr. Technol., 100(3), 404-417.
- Amontons, G. (1699) "De la resistance caus'ee dans les machines," Memoires de l'Academie Royal, A, Chez Gerard Kuyper, Amsterdam, 1706, 257-282.
- Anderson, W. J. (1978) "The Practical Impact of Elastohydrodynamic Lubrication," Proceedings of Fifth Leeds-Lyon Symposium on Tribology on 'Elastohydrodynamics and Related Topics', D. Dowson, C. M. Taylor, M. Godet, and D. Berthe, eds., Mechanical Engineering Publications, Ltd., 217-226.
- Anderson, W. J. and Zaretsky, E. V. (1968) "Rolling-Element Bearings." Mach. Des. (Bearing Reference Issue), 40(14), 22-39.
- Anderson, W. J. and Zaretsky, E. V. (1973) "Rolling-Element Bearings - A Review of the State of the Art," Tribology Workshop sponsored by National Science Foundation, Atlanta, Ga., Oct. 19-20, 1972.

- Archard, J. F. (1968) "Non-Dimensional Parameters in Isothermal Theories of Elastohydrodynamic Lubrication." J. Mech. Eng. Sci., 10(2), 165-167.
- Archard, J. F. and Cowking, E. W. (1965-66) "Elastohydrodynamic Lubrication at Point Contacts," Proc. Inst. Mech. Eng., London, 180(3B), 47-56.
- Archard, J. F. and Kirk, M. T. (1961) "Lubrication at Point Contacts" Proc. R. Soc. London, Ser. A, 261, 532-550.
- Archard, J. F. and Kirk, M. T. (1964) "Film Thickness for a Range of Lubricants Under Severe Stress," J. Mech. Eng. Sci., 6, 101-102.
- Ausherman, V. K., Nagaraj, H. S., Sanborn, D. M., and Winer, W. O. (1976) "Infrared Temperature Mapping in Elastohydrodynamic Lubrication," J. Lubr. Technol., 98(2), 236-243.
- Baglin, K. P. and Archard, J. F. (1972) "An Analytic Solution of the Elastohydrodynamic Lubrication of Materials of Low Elastic Modulus," Proceedings of Second Symposium on Elastohydrodynamic Lubrication, Institution of Mechanical Engineers, London, 13.
- Bair, S. and Winer, W. (1979) "Shear Strength Measurements of Lubricants at High Pressures," J. Lubr. Technol. 101(3), 251-257.
- Bamberger, E. N. (1967) "The Effect of Ausforming on the Rolling Contact Fatigue Life of a Typical Bearing Steel," J. Lubr. Technol., 89(1), 63-75.
- Bamberger, E. N. (1972) "The Thermomechanical Working of Electro-Slag Melted M-50 Bearing Steel," R72AEG290, General Electric Co., Cincinnati, Ohio.
- Bamberger, E. N., Harris, T. A., Kacmarsky, W. M., Moyer, C. A., Parker, R. J., Sherlock, J. J., and Zaretsky, E. V. (1971) Life Adjustment Factors for Ball and Roller Bearings. American Society of Mechanical Engineers, New York.

- Bamberger, E. N., Zaretsky, E. V., and Singer, H. (1976) "Endurance and Failure Characteristics of Main-Shaft Jet Engine Bearing at 3×10^6 DN," J. Lubr. Technol., 98(4), 580-585.
- Barus, C. (1893) "Isotherms, Isopiestic, and Isometrics Relative to Viscosity," Am. J. Sci., 45, 87-96.
- Barwell, F. T. (1974) "The Tribology of Wheel on Rail," Tribol. Int., 7, (4), 146-150.
- Barwell, F. T. (1979) "Bearing Systems - Principles and Practice," Oxford University Press, Oxford.
- Bell, J. C. and Kannel, J. W. (1970) "Simulation of Ball-Bearing Lubrication with a Rolling-Disk Apparatus," J. Lubr. Technol., 92, 1-15.
- Bell, J. C., Kannel, J. W., and Allen, C. M. (1964) "The Rheological Behaviour of the Lubricant in the Contact Zone of a Rolling Contact System," J. Basic Eng., 86(3), 423-432.
- Bisson, E. E. and Anderson, W. J. (1964) "Advanced Bearing Technology," NASA SP-38.
- Biswas, S. and Snidle, R. W. (1976) "Elastohydrodynamic Lubrication of Spherical Surfaces of Low Elastic Modulus," J. Lubr. Technol., 98(4), 524-529.
- Blok, H. (1952) Discussion of paper by E. McEwen. Gear Lubrication Symposium. Part I. The Lubrication of Gears, J. Inst. Petrol., 38, 673.
- Blok, H. (1965) "Inverse Problems in Hydrodynamic Lubrication and Design Directives for Lubricated Flexible Surfaces," Proceedings of International Symposium on Lubrication and Wear, D. Muster and B. Sternlicht, eds., McCutchan, Berkeley, 1-151.

- Brewe, D. E., Coe, H. H., and Scibbe, H. W. (1969) "Cooling Studies with High-Speed Ball Bearings Operating in Cool Hydrogen Gas," Trans. ASLE, vol. 12, 66-76.
- Brewe, D. E. and Hamrock, B. J. (1977) "Simplified Solution for Elliptical-Contact Deformation Between Two Elastic Solids," J. Lubr. Technol. 99(4), 485-487.
- Brewe, D. E., Hamrock, B. J., and Taylor, C. M. (1979) "Effect of Geometry on Hydrodynamic Film Thickness," J. Lubr. Technol., 101(2), 231-239.
- Brown, P. F. and Potts, J. R. (1977) "Evaluation of Powder Processed Turbine Engine Ball Bearings," PWA-FR-8481, Pratt & Whitney Aircraft Group, West Palm Beach, Fla. (AFAPL-TR-77-26.)
- Cameron, A. (1954) "Surface Failure in Gears," J. Inst. Petrol., vol. 40, 191.
- Cameron, A. (1966) The Principles of Lubrication, Wiley, New York.
- Cameron, A. and Gohar, R. (1966) "Theoretical and Experimental Studies of the Oil Film in Lubricated Point Contact," Proc. R. Soc. London, Ser. A., 291, 520-536.
- Carburi, M. (1777) "Monument Elevé a la Gloire de Pierre-le-Grand, ou Relation Des Travaux et des Moyens Mechaniques Qui ont été employés pour transporter à Petersbourg un Rocher de trois millions pesant, destiné à servir de base à la Statue équestre de cet Empereur; avec un Examen Physique et Chymique de meme Rocher," Paris, (Bookseller: Nyon aîné, Libraire, rue Saint-Lean-de-Beauvois; Printer: Imprimeur-Libraire, rue de la Harpe, vis-à-vis la rue S. Severin).

- Castle, P. and Dowson, D. (1972) "A Theoretical Analysis of the Starved Contact," Proceedings of Second Symposium on Elastohydrodynamic Lubrication, Institution of Mechanical Engineers, London, 131.
- Cheng, H. S. (1967) "Calculation of Elastohydrodynamic Film Thickness in High-Speed Rolling and Sliding Contacts," Mechanical Technology Technical Report MTI-67TR24, May 1967.
- Cheng, H. S. (1970) "A Numerical Solution to the Elastohydrodynamic Film Thickness in an Elliptical Contact," J. Lubr. Technol., 92(1), 155-162.
- Cheng, H. S. and Orcutt, F. K. (1965-66) "A Correlation Between the Theoretical and Experimental Results on the Elastohydrodynamic Lubrication of Rolling and Sliding Contacts," Elastohydrodynamic Lubrication, Symposium, Leeds, England, Sept. 21-23, 1965, General Papers. Institution of Mechanical Engineers, London, 111-121.
- Cheng, H. S. and Sternlicht, B. (1964) "A Numerical Solution for the Pressure, Temperature, and Film Thickness Between Two Infinitely Long, Lubricated Rolling and Sliding Cylinders, Under Heavy Loads," J. Basic Eng. 87(3), 695-707.
- Chiu, Y. P. (1974) "An Analysis and Prediction of Lubricant Film Starvation in Rolling Contact Systems," ASME Trans., 17(1), 22-35.
- Clark, R. H. (1938) "Earliest Known Ball Thrust Bearing Used in Windmill," English Mechanic, 30 (Dec.) 223.
- Coulomb, C. A. (1785) "Théorie des Machines Simples, en ayant égard au frottement de leur parties, et a la roideur des cordages," Academic Royale des Sciences, Mem. Math. Phys., X, Paris, 161-342.

- Crook, A. W. (1957) "Simulated Gear-Tooth Contact: Some Experiments Upon Their Lubrication and Sub-Surface Deformation," Proc. Inst. Mech. Eng., London, 171, 187.
- Crook, A. W. (1958) "The Lubrication of Rollers, I," Phil. Trans. R. Soc. London, Ser. A, 250, 387-409.
- Crook, A. W. (1961) "Elasto-Hydrodynamic Lubrication of Rollers, Nature," 190, 1182.
- Crook, A. W. (1963) "The Lubrication of Rollers, IV - Measurements of Friction and Effective Viscosity," Phil. Trans. R. Soc. London, Ser. A, 255, 281-312.
- Dalmaz, G. and Godet, M. (1973) "Traction, Load, and Film Thickness in Lightly Loaded Lubricated Point Contacts," J. Mech. Eng. Sci., 15(6), 400-409.
- Dalmaz, G. and Godet, M. (1978) "Film Thickness and Effective Viscosity of Some Fire Resistant Fluids in Sliding Point Contacts," J. Lubr. Technol., 100(2), 304-308.
- Denhard, W. G. (1966) "Cost Versus Value of Ball Bearings," Gyro-Spin Axis Hydrodynamic Bearing Symposium, Vol. II, Ball Bearings. Massachusetts Institute of Technology, Cambridge, Mass., Tab. 1.
- Desaguliers, J. T. (1734) A Course of Experimental Philosophy, 2 Volumes, London, Volume I, with 32 copper plates.
- Dowson, D. (1962) "A Generalized Reynolds Equation for Fluid-Film Lubrication," Int. J. Mech. Sci., 4, 159-170.
- Dowson, D. (1965) "Elastohydrodynamic Lubrication - An Introduction and a Review of Theoretical Studies," Institute of Mechanical Engineers, London, Paper R1, 7-15.

- Dowson, D. (1968) "Elastohydrodynamics," Proc. Inst. Mech. Eng., London, 182(3A), 151-167.
- Dowson, D. (1975) "The Inlet Boundary Condition," Cavitation and Related Phenomena in Lubrication. D. Dowson, M. Godet, and C. M. Taylor, eds., Mechanical Engineering Publications, Ltd., New York, 143-152.
- Dowson, D. (1976) "The Origins of Rolling Contact Bearings," T. Sakuri, ed., Proceedings of JSLE-ASLE International Lubrication Conference, Elsevier, Amsterdam, 20-38.
- Dowson, D. (1979) History of Tribology, Longman, London and New York.
- Dowson, D. (1981) "Lubrication of Joints," Chapter 13 in "The Biomechanics of Joints and Joint Replacements," Edited by D. Dowson and V. Wright, Mechanical Engineering Publications, Bury St. Edmunds, Suffolk. (To be published.)
- Dowson, D. and Hamrock, B. J. (1976) "Numerical Evaluation of the Surface Deformation of Elastic Solids Subjected to a Hertzian Contact Stress," ASLE Trans., 19(4), 279-286.
- Dowson, D. and Higginson, G. R. (1959) "A Numerical Solution to the Elastohydrodynamic Problem," J. Mech. Eng. Sci., 1(1), 7-15.
- Dowson, D. and Higginson, G. R. (1961) "New Roller-Bearing Lubrication Formula," Engineering London, vol. 192, 158.
- Dowson, D. and Higginson, G. R. (1964), "A Theory of Involute Gear Lubrication," Institute of Petroleum Gear Lubrication; Proceedings of a Symposium organized by the Mechanical Tests of Lubricants Panel of the Institute, (1964), Elsevier, 8-15.
- Dowson, D. and Higginson, G. R. (1966) Elastohydrodynamic Lubrication, The Fundamentals of Roller and Gear Lubrication. Pergamon, Oxford.

- Dowson, D., Saman, W. Y., and Toyoda, S. (1979) "A Study of Starved Elastohydrodynamic Line Contacts," Proceedings of Fifth Leeds-Lyon Symposium on Tribology on 'Elastohydrodynamics and Related Topics,' D. Dowson, C. M. Taylor, M. Godet, and D. Berthe, eds., Mechanical Engineering Publications, Ltd., 92-103.
- Dowson, D. and Swales, P. D. (1969) "The Development of Elastohydrodynamic Conditions in a Reciprocating Seal," Proceedings of Fourth International Conference on Fluid Sealing, Vol. 2, Paper 1, British Hydromechanics Research Association, 1-9.
- Dowson, D. and Toyoda, S. (1979) "A Central Film Thickness Formula for Elastohydrodynamic Line Contacts." Proceedings of Fifth Leeds-Lyon Symposium on Tribology on 'Elastohydrodynamics and Related Topics,' D. Dowson, C. M. Taylor, M. Godet, and D. Berthe, eds., Mechanical Engineering Publications, Ltd., 104-115.
- Dowson, D. and Whitaker, A. V. (1965-66) "A Numerical Procedure for the Solution of the Elastohydrodynamic Problems of Rolling and Sliding Contacts Lubricated by a Newtonian Fluid," Proc. Inst. Mech. Eng., London, 180(3B), 57.
- Dyson, A. (1970) "Flow Properties of Mineral Oils in Elastohydrodynamic Lubrication," Phil. Trans. R. Soc. London, Ser. A, 258(1093), 529-564.
- Dyson, A., Naylor, H., and Wilson, A. R. (1965-66) "The Measurement of Oil-Film Thickness in Elastohydrodynamic Contacts," Proceedings of Symposium on Elastohydrodynamic Lubrication, Leeds, England, Institution of Mechanical Engineers, London, 76-91.
- Dupuit, A. J. E. J. (1839), "Résumé de Mémoire sur le tirage des voitures et sur le frottement de seconde espee," Competes rendus de l'Académie des Sciences, Paris, IX, 689-700, 775.

- Eaton, J. T. H., ed. (1969) "A Trip Down Memory Lane," The Dragon, XLIV (5), 5-7.
- ESDU (1965) "General Guide to the Choice of Journal Bearing Type," Engineering Sciences Data Unit, Item 65007, Institute of Mechanical Engineers, London.
- ESDU (1967) "General Guide to the Choice of Thrust Bearing Type," Engineering Sciences Data Unit, Item 67033, Institution of Mechanical Engineers, London.
- ESDU (1978) "Grease Life Estimation in Rolling Bearings," Engineering Sciences Data Unit, Item 78032, Institution of Mechanical Engineers, London.
- ESDU (1978) "Contact Phenomena. I: Stresses, Deflections and Contact Dimensions for Normally-Loaded Unlubricated Elastic Components," Engineering Sciences Data Unit, Item 78035, Institution of Mechanical Engineers, London.
- Evans, H. P., Biswas, S., and Snidle, R. W. (1978) "Numerical Solution of Isothermal Point Contact Elastohydrodynamic Lubrication Problems," Proceeding of First International Conference on Numerical Methods in Laminar and Turbulent Flow, Pentech Press, London, 639-656.
- Evans, H. P. and Snidle, R. W. (1978) "Toward a Refined Solution of the Isothermal Point Contact EHD Problem." International Conference Fundamentals of Tribology, Massachusetts Institute of Technology, Cambridge, Mass., June 19-22, 1978.
- Fein, R. S. (1968) Discussion on the Papers of J. K. Appeldorn and A. B. Metzner, J. Lubr. Technol., 90, 540-542.

- Fellows, T. G., Dowson, D., Perry, F. G., and Plint, M. A. (1963)
 "Perbury Continuously Variable Ratio Transmission," in N. A. Carter, Ed.
Advances in Automobile Engineering, Part 2; Pergamon Press, 123-139.
- Foord, C. A., Hammann, W. C., and Cameron, A. (1968) "Evaluation of
 Lubricants Using Optical Elastohydrodynamics," ASLE Trans., 11, 31-43.
- Foord, C. A., Wedeven, L. D., Westlake, F. J. and Cameron, A. (1969-70)
 "Optical Elastohydrodynamics," Proc. Inst. Mech. Eng., London, Part I,
 184, 487-503.
- Fromm, H. (1948), "Laminare Strömung Newtonscher und Maxwellscher
 Flüssigkeiten," Angew Math. Mech., 28(2), 43-54.
- Furey, M. J. (1961) "Metallic Contact and Friction Between Sliding
 Surfaces," ASLE Trans., vol. 4, 1-11.
- Gentle, C. R. and Cameron, A. (1973) "Optical Elastohydrodynamics at
 Extreme Pressure," Nature, 246(5434), 478-479.
- Gohar, R. and Cameron A. (1966) "The Mapping of Elastohydrodynamic
 Contacts," ASLE Trans., 10, 215-225.
- Goodman, J. (1912) "(1) Roller and Ball Bearings;" "(2) The Testing of
 Antifriction Bearing Materials," Proceedings of the Institute of Civil
 Engineers, CLXXXIX, Session 1911-12, Pt. III, pp. 4-88.
- Greenwood, J. A. (1969) "Presentation of Elastohydrodynamic Film-Thickness
 Results." J. Mech. Eng. Sci., 11(2), 128-132.
- Greenwood, J. A. and Kauzlarich, J. J. (1973) "Inlet Shear Heating in
 Elastohydrodynamic Lubrication," J. Lubr. Technol., 95(4), 401-416.

- Grubin, A. N. (1949) "Fundamentals of the Hydrodynamic Theory of Lubrication of Heavily Loaded Cylindrical Surfaces," Investigation of the Contact Machine Components. Kh. F. Ketova, ed., Translation of Russian Book No. 30, Central Scientific Institute for Technology and Mechanical Engineering, Moscow, Chapter 2. (Available from Dept. of Scientific and Industrial Research, Great Britain, Transl. CTS-235, and from Special Libraries Association, Chicago, Trans. R-3554.)
- Gunther, R. T. (1930), Early Science in Oxford, Volumes VI and VII, "The Life and Work of Robert Hooke," Vol. VII, Pt. II, 666-679, printed for the author at the Oxford University Press by John Johnson (Oxford).
- Hall, L. F. (1957) "A Review of the Papers on the Lubrication of Rotating Bearings and Gears," Proceedings of Conference on Lubrication and Wear, Institution of Mechanical Engineers, pp. 425-429.
- Hamilton, G. M. and Moore, S. L. (1971) "Deformation and Pressure in an Elastohydrodynamic Contact," Proc. R. Soc., London, Ser. A, 322, 313-330.
- Halling, J. (1976) Introduction of Tribology, Wykeham Publ., London.
- Hamrock, B. J. (1976) Elastohydrodynamic Lubrication of Point Contacts, Ph.D. Dissertation, University of Leeds, Leeds, England.
- Hamrock, B. J. and Anderson, W. J. (1973) "Analysis of an Arched Outer-Race Ball Bearing Considering Centrifugal Forces," J. Lubr. Technol., 95(3), 265-276.
- Hamrock, B. J. and Dowson, D. (1974) "Numerical Evaluation of Surface Deformation of Elastic Solids Subjected to Hertzian Contact Stress," NASA TN D-7774.
- Hamrock, B. J. and Dowson, D. (1976a) "Isothermal Elastohydrodynamic Lubrication of Point Contacts, Part I - Theoretical Formulation," J. Lubr. Technol., 98(2), 223-229.

- Hamrock, B. J. and Dowson, D. (1976b) "Isothermal Elastohydrodynamic Lubrication of Point Contacts, Part II - Ellipticity Parameter Results," J. Lubr. Technol., 98(3), 375-378.
- Hamrock, B. J. and Dowson, D. (1977a) "Isothermal Elastohydrodynamic Lubrication of Point Contacts, Part III - Fully Flooded Results," J. Lubr. Technol., 99(2), 264-276.
- Hamrock, B. J. and Dowson, D. (1977b) "Isothermal Elastohydrodynamic Lubrication of Point Contacts, Part IV - Starvation Results," J. Lubr. Technol., 99(1), 15-23.
- Hamrock, B. J. and Dowson, D. (1978) "Elastohydrodynamic Lubrication of Elliptical Contacts for Materials of Low Elastic Modulus, Part I - Fully Flooded Conjunction," J. Lubr. Technol., 100(2), 236-245.
- Hamrock, B. J. and Dowson, D. (1979a) "Elastohydrodynamic Lubrication of Elliptical Contacts for Materials of Low Elastic Modulus, Part II - Starved Conjunction," J. Lubr. Technol., 101(1), 92-98.
- Hamrock, B. J. and Dowson, D. (1979b) "Minimum Film Thickness in Elliptical Contacts for Different Regimes of Fluid-Film Lubrication," Proceedings of Fifth Leeds-Lyon Symposium on Tribology on 'Elastohydrodynamics and Related Topics,' D. Dowson, C. M. Taylor, M. Godet, and D. Berthe, eds., Mechanical Engineering Publications, Ltd., 22-27.
- Hardy, W. B. and Doubleday, I. (1922a) "Boundary Lubrication - the Temperature Coefficient," Proc. R. Soc. London, Ser. A, 101, 487-492.
- Hardy, W. B. and Doubleday, I. (1922b) "Boundary Lubrication - the Paraffin Series," Proc. R. Soc. London, Ser. A, 100, 550-574.
- Harris, T. A. (1966) Rolling Bearing Analysis. Wiley, New York.

- Harris, T. A. (1971) "An Analytical Method to Predict Skidding in Thrust-Loaded, Angular-Contact Ball Bearings," J. Lubr. Technol., vol. 93, 17-24.
- Harrison, H. C. (1949) The Story of Sprowston Mill, Phoenix House, London.
- Harrison, W. J. (1913) "The Hydrodynamical Theory of Lubrication with Special Reference to Air as a Lubricant," Trans. Cambridge Philos. Soc., xxii (1912-25), 6-54.
- Harrison, G. and Trachman, E. G. (1972) "The Role of Compressional Viscoelasticity in the Lubrication of Rolling Contacts," J. Lubr. Technol., 94, 306-312.
- Heathcote, H. L. (1921) "The Ball Bearing: In the Making, Under Test, and on Service," Proc. Instn. Automotive Engrs., London, 15, pp. 569-702.
- Herrebrugh, K. (1968) "Solving the Incompressible and Isothermal Problem in Elastohydrodynamic Lubrication Through an Integral Equation," J. Lubr. Technol., 90(1), 262-270.
- Hersey, M. D. (1966) Theory and Research in Lubrication - Foundations for Future Developments, Wiley, New York.
- Hersey, M. S. and Hopkins, R. F. (1954) "Viscosity of Lubricants Under Pressure. Coordinated Data from Twelve Investigations." ASME, New York.
- Hertz, H. (1881) "The Contact of Elastic Solids," J. Reine Angew. Math., 92, 156-171.
- Hooke, C. J. (1977) "The Elastohydrodynamic Lubrication of Heavily Loaded Contacts," J. Mech. Eng. Sci., 19(4), 149-156.
- Hirst, W. and Moore, A. J. (1974) "Non-Newtonian Behavior in Elastohydrodynamic Lubrication," Proc. R. Soc. London, Ser. A, 337, 101-121.

- Houghton, P. S. (1976) Ball and Roller Bearings, Applied Science Publishers, Ltd., London.
- Jacobson, B. (1970) "On the Lubrication of Heavily Loaded Spherical Surfaces Considering Surface Deformations and Solidification of the Lubricant." Acta Polytech. Scand., Mech. Eng. Ser. No. 54.
- Jacobson, B. (1972) "Elasto-Solidifying Lubrication of Spherical Surfaces." American Society of Mechanical Engineers Paper No. 72-LUB-7.
- Jacobson, B. (1973) "On the Lubrication of Heavily Loaded Cylindrical Surfaces Considering Surface Deformations and Solidification of the Lubricant," J. Lubr. Technol., 95(3), 321-27.
- Jamison, W. E., Lee, C. C., and Kauzlarich, J. J. (1978) "Elasticity Effects on the Lubrication of Point Contacts," ASLE Trans., 21(4), 299-306.
- Johnson, B. L. (1964) "A 'Stainless High Speed' Steel for Aerospace Applications," Metal Prog., 86(3), 116-118.
- Johnson, B. L. (1965) "High Temperature Wear Resisting Steel," U.S. Patent No. 3,167,423, Jan. 1965.
- Johnson, K. L. (1970) "Regimes of Elastohydrodynamic Lubrication." J. Mech. Eng. Sci., 12(1), 9-16.
- Johnson, K. L. and Cameron, R. (1967) "Shear Behavior of Elastohydrodynamic Oil Films at High Rolling Contact Pressures," Proc. Ins. Mech. Eng., Part 1, 182, 307-319.
- Johnson, K. L. and Roberts, A. D. (1974) "Observation of Viscoelastic Behaviour of an Elastohydrodynamic Lubricant Film," Proc. R. Soc. London, Ser. A, 337, 217-242.
- Johnson, K. L. and Tevaarwerk, J. L. (1977) "Shear Behaviour of Elastohydrodynamic Oil Films," Proc. R. Soc. London, Ser. A, 356, 215-236.

- Jones, A. B. (1946) "Analysis of Stresses and Deflections," New Departure Engineering Data, General Motors Corp., Bristol, Conn.
- Jones, A. B. (1956) "The Mathematical Theory of Rolling-Element Bearings," Mechanical Design and Systems Handbook.
- Kannel, J. W., Bell, J. C., and Allen, C. M. (1964) "Methods for Determining Pressure Distribution in Lubricated Rolling Contact," ASLE Paper 64-LC-23, Presented at ASME-ASLE Lubrication Conference, Washington, D.C., Oct. 13-16, 1964.
- Kakuta, K. (1979) "The State of the Art of Rolling Bearings in Japan," Bull. Japan Soc. Prec. Eng., 13(4), 169-176.
- Kapitza, P. L. (1955) "Hydrodynamic Theory of Lubrication During Rolling," Zh. Tekh. Fiz., 25(4), 747-762.
- Koye, K. A. and Winer, W. O. (1980) "An Experimental Evaluation of the Hamrock and Dowson Minimum Film Thickness Equation for Fully Flooded EHD Point Contacts," International ASME/ASLE Lubrication Conference, San Francisco, August 1980.
- Kunz, R. K. and Winer, W. O. (1977) Discussion 275-276, to Hamrock, B. J. and Dowson, D. "Isothermal Elastohydrodynamic Lubrication of Point Contacts, Part III - Fully Flooded Results," J. Lubr. Technol., 99(2), 264-275.
- Lane, T. B. (1951) "Scuffing Temperatures of Boundary Lubricant Films," Br. J. Appl. Phys., 2, (Suppl. 1), 35-38.
- Lane, T. B. and Hughes, J. R. (1952) "A Study of the Oil Film Formation in Gears by Electrical Resistance Measurements," Br. J. Appl. Phys., 3(10), 315-318.
- Lamb, H. (1932) Hydrodynamics. Cambridge University Press.

- Layard, A. H. (1849) Nineveh and Its Remains, Vols. I and II, John Murray, London.
- Layard, A. H. (1853) Discoveries in the Ruins of Nineveh and Babylon, Vols. I and II, John Murray, London.
- Lee, D., Sanborn, D. M., and Winer, W. O. (1973) "Some Observations of the Relationship Between Film Thickness and Load in High Hertz Pressure Sliding Elastohydrodynamic Contacts," J. Lubr. Technol., 95(3), 386.
- Leibnitz, G. W. (1706) "Tentamen de natura et remedlie resistenziarum in machines," Miscellanea Berolinensia. Class. mathem, 1710, (Jean Boudot, Paris), 1, 307.
- Lewicki, W. (1955) "Some Physical Aspects of Lubrication in Rolling Bearings and Gears," Engineer, 200 (5193), 176-178, and (5194), 212-215.
- Lundberg, G. and Palmgren, A. (1947) "Dynamic Capacity of Rolling Bearings," Acta Polytech., Mech. Eng. Sci., 1(3).
- Martin, H. M. (1916) "Lubrication of Gear Teeth," Engineering, London, 102, 199.
- McEwen, E. (1952) "The Effect of Variation of Viscosity with Pressure on the Load Carrying Capacity of Oil Films Between Gear Teeth," J. Inst. Petrol., 38, 646.
- Meldahl, A. (1941) "Contribution to the Theory of the Lubrication of Gears and of the Stressing of the Lubricated Flanks of Gear Teeth," Brown Boveri Review, 28(11), 374.
- Merritt, H. E. (1935) "Worm-Gear Performance," Proc. Inst. Mech. Eng., London, 129, 127-158.
- Meyer, D. R. and Wilson, C. C. (1971) "Measurement of Elastohydrodynamic Oil Film Thickness and Wear in Ball Bearings by the Strain Gage Method," J. Lubr. Technol., 93(2), 224-230.

- Moes, H. (1965-66) "Communication, Elastohydrodynamic Lubrication," Proc. Inst. Mech. Eng., London, 180(3B), 244-245.
- Moes, H. and Bosma, R. (1972) "Film Thickness and Traction in EHL at Point Contact," Proceedings of Second Symposium on Elastohydrodynamic Lubrication, Leeds, England, Institution of Mechanical Engineers, London, 149.
- Moore, A. J. (1973) "Non-Newtonian Behaviour in Elastohydrodynamic Lubrication," Ph.D. Thesis, University of Reading.
- Morgan, M. H. and Warren, H. L. (1960) Translation of Vitruvius: The Ten Books of Architecture, Dover, New York.
- Morin, A. J. (1835) "Nouvelles expériences faites à Metz en 1833 sur le frottement, sur la transmission due mouvement par le choc, sur le résistance des milieun imparfaits a le pénétration des projectiles, et sur le frottement pendant le choc," Mem. Savans Etrang. (Paris), VI, 641-785; Ann. Min. X, (1836), 27-56.
- Nagaraj, H. S., Sanborn, D. M., and Winer, W. O. (1977) "Effects of Load, Speed, and Surface Roughness on Sliding EHD Contact Temperature," J. Lubr. Technol., 99(4), 254-263.
- Navier, C. L. M. H. (1823) "Memoire sur les lois du mouvement des fluides," Mem. Acad. R. Sci., 6(2), 389-440.
- Needham, J. (1965) Science and Civilization in China, Vol. 4, Physics and Physical Technology, Part II, Mechanical Engieering, Cambridge University Press.

- Newton, I. (1687) Philosophiae Naturales Principia Mathematica, Imprimature S. Pepys, Reg. Soc. Praeses, 5 Julii 1686. Revised and supplied with a historical and explanatory appendix by F. Cajori, edited by R. T. Crawford (1934), and published by the University of California Press, Berkeley and Los Angeles (1966).
- Orcutt, F. K. and Cheng, H. S. (1966) "Lubrication of Rolling-Contact Instrument Bearings," Gyro-Spin Axis Hydrodynamic Bearing Symposium, Vol. 2, Ball Bearings, Massachusetts Institute of Technology, Cambridge, Mass., Tab. 5.
- Pai, S. I (1956) Viscous Flow Theory, Vol. I - Laminar Flow. Van Nostrand Reinhold, New Jersey.
- Palmgren, A. (1945) "Ball and Roiller Bearing Engineering," S. K. F. Industries, Philadelphia.
- Parker, R. J. and Hodder, R. S. (1978) "Roller-Element Fatigue Life of AMS 5749 Corrosion Resistant, High Temperature Bearing Steel," J. Lubr. Technol., 100(2), 226-235.
- Parker, R. J. and Kannel, J. W. (1971) "Elastohydrodynamic Film Thickness Between Rolling Disks with a Synthetic Paraffinic Oil to 589 K (600° F); NASA TN D-6411.
- Parker, R. J. and Zaretsky, E. V. (1978) "Rolling-Element Fatigue Life of AISI M-50 and 18-4-1 Balls." NASA TP-1202.
- Peppler, W. (1936) "Untersuchunge uber die Druckubertragung bei Balasteten und Geschmierten um Laufenden Achsparallelen Zylinder," Maschinenelemente-Tagung Archen 1935, 42; V. D. I. Verlag, Berlin, 1936.
- Peppler, W. (1938) "Druckubertragung an Geschmeirten Zylindriachen Gleit und Wälzflächen," V. D. I. Forschungshaft, 391.

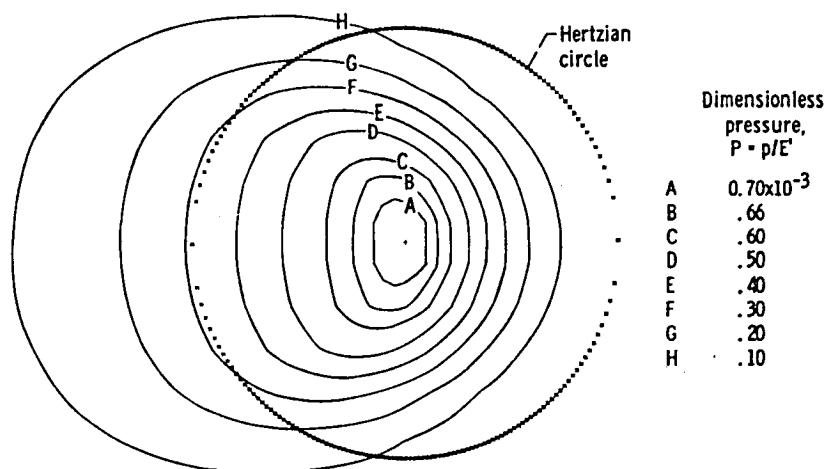
- Petrov, N. P. (1883) "Friction in Machines and the Effect of the Lubricant," Inzh. Zh., St. Peterb., 1, 71-140; 2, 227-279; 3, 377-436; 4, 535-564.
- Petrusevich, A. S. (1951) "Fundamental Conclusion from the Contact-Hydrodynamic Theory of Lubrication," dzo. Akad. Nauk. SSSR (OTN), 2, 209.
- Piggott, S. (1968) "The Earliest Wheeled Vehicles and the Caucasian Evidence," Proc. Prehist. Soc., XXXIV, (8), 266-318.
- Pirvics, J. (1980) "Numerical Analysis Techniques and Design Methodology for Rolling Element Bearing Load Support Systems," in International Conference on Bearing Design: Historical Aspects, Present Technology and Future Problems; Century 2 - Emerging Technology, W. J. Anderson, ed., American Society of Mechanical Engineers, New York, 1980, 47-85.
- Plint, M. A. (1967) "Traction in Elastohydrodynamic Contact," Proc. Inst. Mech. Eng., London, Part 1, 182(14), 300-306.
- Poritsky, H., Hewlett, C. W., Jr., and Coleman, R. E., Jr. (1947) "Sliding Friction of Ball Bearings of the Pivot Type," J. Appl. Mech., 14(4), 261-268.
- Pritchard, C. (1981) "Traction Between Rolling Steel Surfaces - A Survey of Railway and Laboratory Services," Proceedings of the 7th Leeds-Lyon Symposium on 'Friction and Traction, Leeds, September 1980, Mechanical Engineering Publications. (To be published.)
- Ramelli, A. (1588) "Le Diverse et Artificiose Machine," Paris, France.
- Ranger, A. P., Ettles, C. M. M., and Cameron, A. (1975) "The Solution of Point Contact Elastohydrodynamic Problem," Proc. R. Soc. London, Ser. A, 346, 277-244.
- Reti, L. (1971) "Leonardo on Bearings and Gears," Scientific American, 224, (2), 101-110.

- Reynolds, O. (1875) "On Rolling Friction," Phil. Trans. R. Soc., 166, Pt. 1, 155.
- Reynolds, O. (1886) "On the Theory of Lubrication and Its Application to Mr. Beauchamp Tower's Experiments, Including an Experimental Determination of the Viscosity of Olive Oil," Philos. Trans. R. Soc. London, 177, 157-234.
- Roelands, C. J. A. (1966) Correlational Aspects of the Viscosity-Temperature-Pressure Relationship of Lubricating Oils. Druk. V. R. B., Groningen, Netherlands.
- Rowe, J. (1734) "All Sorts of Wheel-Carriage Improved," printed for Alexander Lyon under Tom's Coffee House in Russell Street, Covent Garden, London.
- Sanborn, D. M. (1969) "An Experimental Investigation of the Elastohydrodynamic Lubrication of Point Contacts in Pure Sliding," Ph.D. Thesis, University of Michigan.
- Schlatter, R. (1974) "Double Vacuum Melting of High Performance Bearing Steels," Ind. Heat. 41(9), 40-55.
- Shaw, M. C. and Macks, E. F. (1949) Analysis and Lubrication of Bearings, McGraw-Hill, New York.
- Sibley, L. B., Bell, J. C., Orcutt, F. K., and Allen, C. M. (1960) "A Study of the Influence of Lubricant Properties on the Performance of Aircraft Gas Turbine Engine Rolling Contact Bearings," WADD Technical Report, 60-189.
- Sibley, L. B. and Orcutt, F. K. (1961) "Elasto-Hydrodynamic Lubrication of Rolling Contact Surfaces," Trans. Amer. Soc. Lub. Engrs., 4(2), 234.
- Smith, F. W. (1959) "Lubricant Behavior in Concentrated Contact Systems - The Caster Oil - Steel System," Wear, 2(4), 250-264.

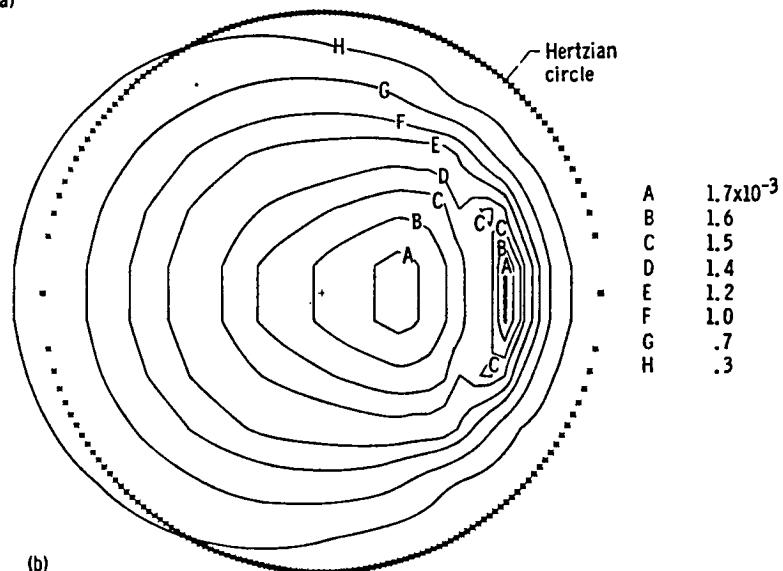
- Smith, F. W. (1962) The Effect of Temperature in Concentrated Contact Lubrication. ASLE Trans. 5(1), 142-148.
- Stokes, G. G. (1845) "On the Theory of Internal Friction of Fluids in Motion," Trans. Cambridge Philos. Soc. 8, 287-319.
- Stribeck, R. (1901) "Kugellager fur beliebige Belastungen," Z. Ver. dt. Ing., 45(3), 73-125.
- Stribeck, R (1907) "Ball Bearings for Various Loads" - translation by H. Hess, Trans. Am. Soc. Mech. Engrs., 29, 420.
- Swingler, C. L. (1980) "Surface Roughness in Elastohydrodynamic Line Contacts," Ph.D. Thesis, University of London (Imperial College).
- Tabor, D. (1962) "Introductory Remarks," in Rolling Contact Phenomena, J. B. Bidwell, ed., Elsevier, Amsterdam, 1-5.
- Tallian, T. E. (1969) "Progress in Rolling Contact Technology," Report AL 690007, SKF Industries, King of Prussia, Pa.
- Tallian, T., Sibley, L., and Valori, R. (1965) "Elastohydrodynamic Film Effects on the Load-Life Behavior of Rolling Contacts," ASMS Paper 65-LubS-11.
- Theyse, F. H. (1966) "Some Aspects of the Influence of Hydrodynamic Film Formation on the Contact Between Rolling/Sliding Surfaces," Wear, 9, 41-59.
- Thorp, N. and Gohar, R. (1972) "Oil Film Thickness and Shape for a Ball Sliding in a Grooved Raceway," J. Lubr. Technol., 94(3), 199-210.
- Timoshenko, S. and Goodier, J. N. (1951) Theory of Elasticity, 2nd ed., McGraw-Hill, New York.

- Trachman, E. G. and Cheng, H. S. (1972) "Thermal and Non-Newtonian Effects on Traction in Elastohydrodynamic Contacts," Proceedings of Second Symposium on Elastohydrodynamic Lubrication, Institution of Mechanical Engineers, London, 142-148.
- Tower, B. (1883) "First Report on Friction Experiments (Friction of Lubricated Bearings)," Proc. Inst. Mech. Eng., London, 632-659.
- Turchina, V., Sanborn, D. M., and Winer, W. O. (1974) "Temperature Measurements in Sliding Elastohydrodynamic Point Contacts," J. Lubr. Technol., 96(3), 464-471.
- Ucelli, G. (1940) "Le Navi Di Nemi," La Libreria Dello Stato, Roma.
- Valori, R. (1978) Discussion to Parker, R. J. and Hodder, R. S. (1978) Rolling-Element Fatigue Life of AMS 5749 Corrosion Resistant, High Temperature Bearing Steel," J. Lubr. Technol., 100(2), 226-235.
- Van Natrus, L., Polly, J., and Van Vuuren, C. (1734 and 1736), Groot Volkomen Moolenbock, 2 Volumes, Amsterdam.
- Varlo, C. (1772) "Reflections Upon Friction with a Plan of the New Machine for Taking It Off in Wheel-Carriages, Windlasses of Ships, etc., Together with Metal Proper for the Machine, the Full Directions for Making It."
- Vaughan, P. (1794) "Axle Trees, Arms, and Boxes," British Patent No. 2006 of A.D. 1794, 1-2, accompanied by 11 diagrams on one sheet.
- Wailes, R. (1954) The English Windmill, Routledge & Kegan Paul, London.
- Wailes, R. (1957) "Windmills" in History of Technology, C. Singer, E. J. Holmyard, A. R. Hall, and T. I. Williams, eds., Volume III, Oxford University Press, pp. 89-109.
- Weber, C. and Saalfeld, K. (1954) Schmierfilm bei Walzen mit Verformung, Zeits ang. Math. Mech. 34 (Nos. 1-2).

- Wedeven, L. E., Evans, D., and Cameron, A. (1971) "Optical Analysis of Ball Bearing Starvation," J. Lubr. Technol., 93(3), 349-363.
- Weibull, W. (1949) "A Statistical Representation of Fatigue Failures in Solids," Trans. Roy. Inst. Technol., (27), Stockholm.
- Whomes, T. L. (1966) The Effect of Surface Quality of Lubricating Film Performance, Ph.D. Dissertation, University of Leeds, Leeds, England.
- Wilcock, D. F. and Booser, E. R. (1957) Bearing Design and Application. McGraw-Hill, New York.
- Willis, T., Seth, B., and Dave, M. (1975) "Evaluation of a Diffraction Method for Thickness Measurement of Oil-Filled Gaps," J. Lubr. Technol. 97(4), 649-650.
- Wilson, A. R. (1979) "The Relative Thickness of Grease and Oil Films in Rolling Bearings," Proc. Inst. Mech. Eng., London, 193(17), 185-192.
- Winn, L. W., Eusepi, M. W., and Smalley, A. J. (1974) "Small, High-Speed Bearing Technology for Cryogenic Turbo-Pumps," MTI-74TR29, Mechanical Technology, Inc., Latham, N.Y. (NASA CR-134615.)
- Wolveridge, P. E., Baglin, K. P., and Archard, J. G. (1971) "The Starved Lubrication of Cylinders in Line Contact," Proc. Inst. Mech. Eng., London, 185(1), 1159-1169.
- Zaretsky, E. V., Anderson, W. J., and Bamberger, E. N. (1969) "Rolling Element Bearing Life for 400° to 600° F." NASA TN D-5002.
- Zaretsky, E. V., Parker, R. J., and Anderson, W. J. (1967) "Component Hardness Differences and Their Effect on Bearing Fatigue," J. Lubr. Technol., 87(1), 47-62.

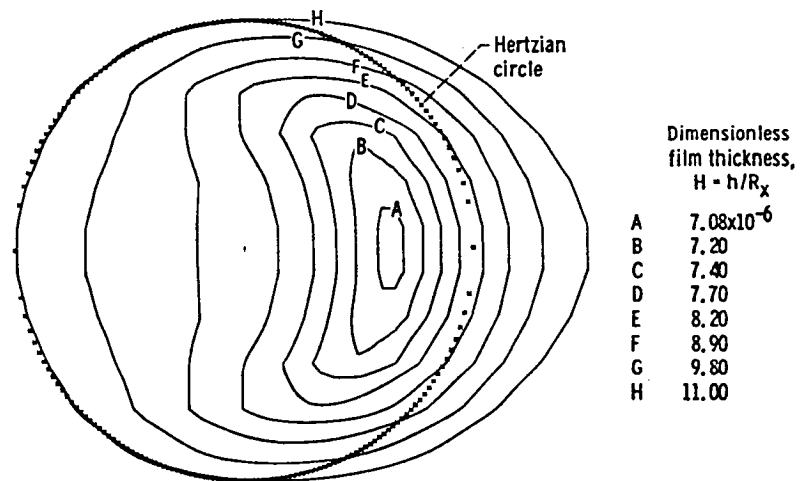


(a)

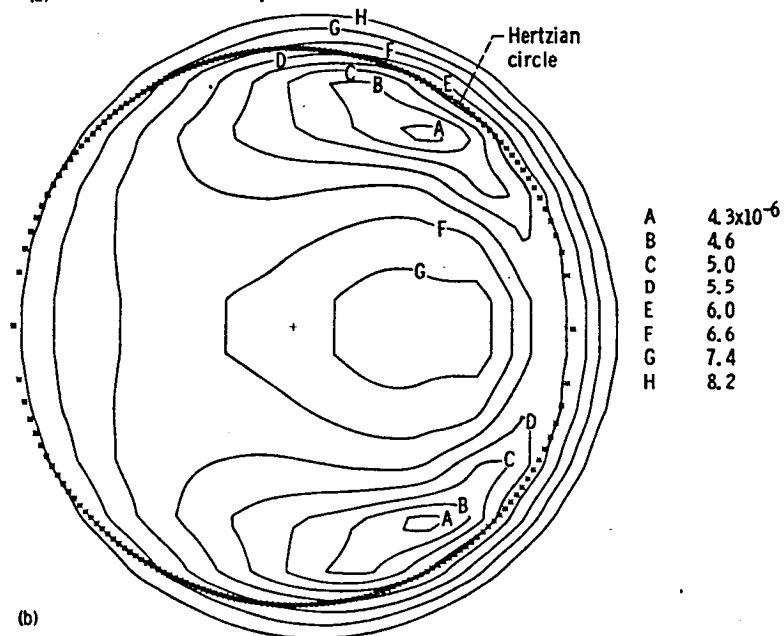


(b)

Figure 8.1. - Contour plots of dimensionless pressure for ellipticity parameters k of 8 and 1.25. The dimensionless parameters U , W , and G are held constant as defined in equation (8.10). (a) $k = 8$. (b) $k = 1.25$.



(a)



(b)

Figure 8.2. - Contour plots of dimensionless film thickness for ellipticity parameters k of 8 and 1.25. The dimensionless parameters U , W , and G are held constant as defined in equation (8.10). (a) $k = 8$. (b) $k = 1.25$.

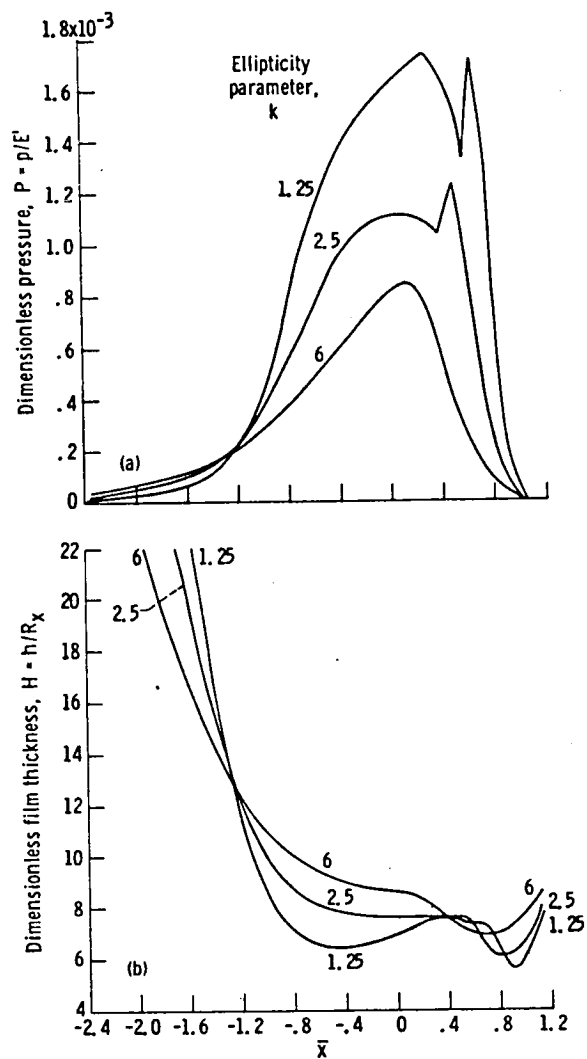
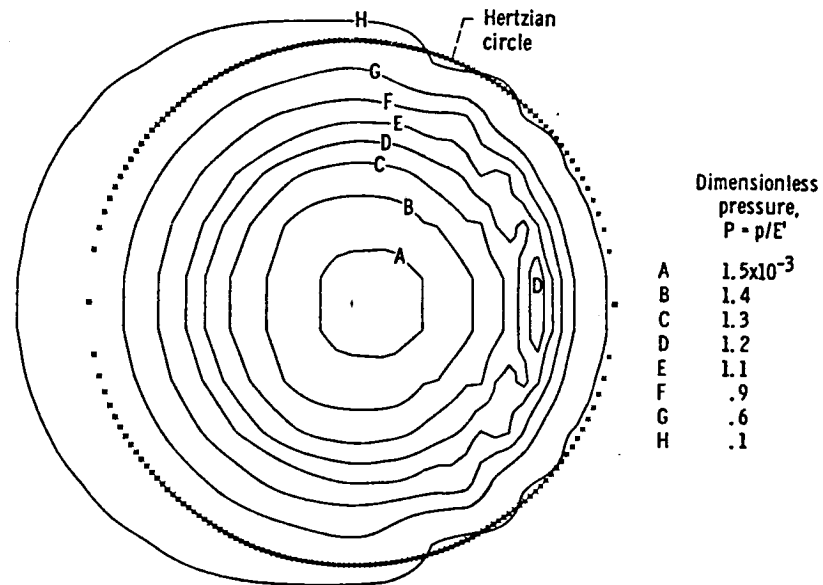
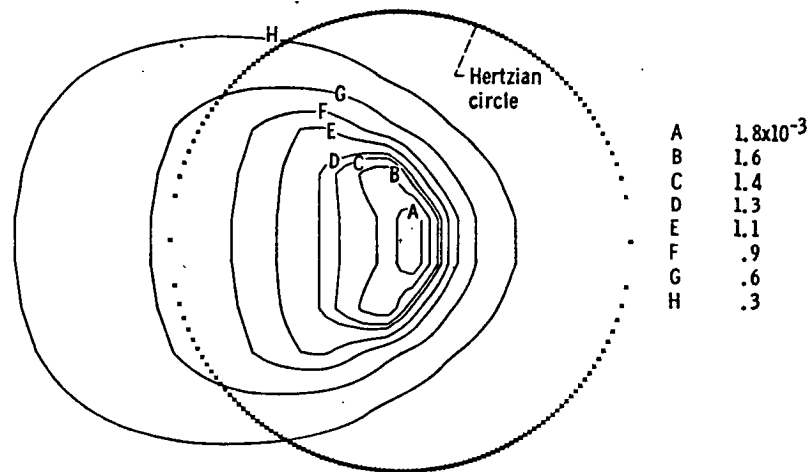


Figure 8.3. - Variation of dimensionless pressure and film thickness on X axis for three values of ellipticity parameter. The value of Y is held fixed near axial center of contact. (a) Dimensionless pressure. (b) Dimensionless film thickness.



(a)



(b)

Figure 8.4. - Contour plots of dimensionless pressure for dimensionless speed parameters U of 0.8416×10^{-12} and 0.5050×10^{-10} . The dimensionless parameters k , W , and G are held constant as defined in equation (8.14). (a) $U = 0.8416 \times 10^{-12}$. (b) $U = 0.5050 \times 10^{-10}$.

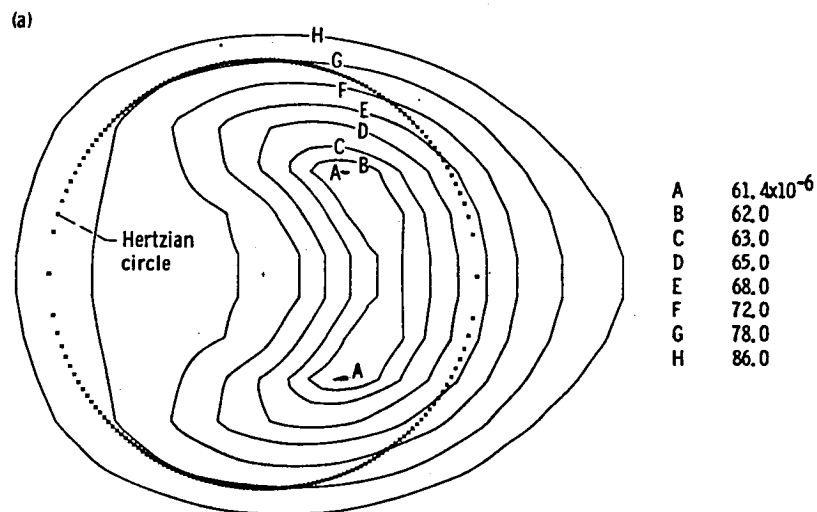
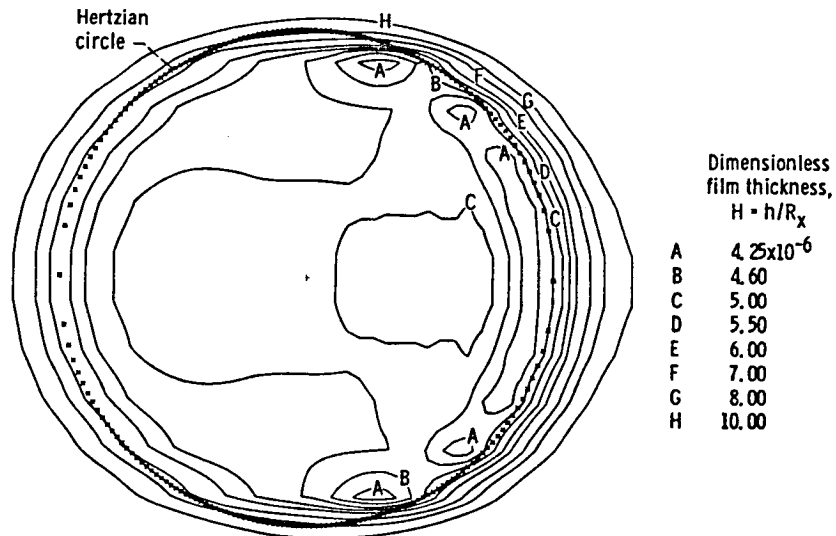


Figure 8.5. - Contour plots of dimensionless film thickness for dimensionless speed parameters U of 0.8416×10^{-12} and 0.5050×10^{-10} . The dimensionless parameters k , W , and G are held constant as defined in equation (8.14). (a) $U = 0.8416 \times 10^{-12}$. (b) $U = 0.5050 \times 10^{-10}$.

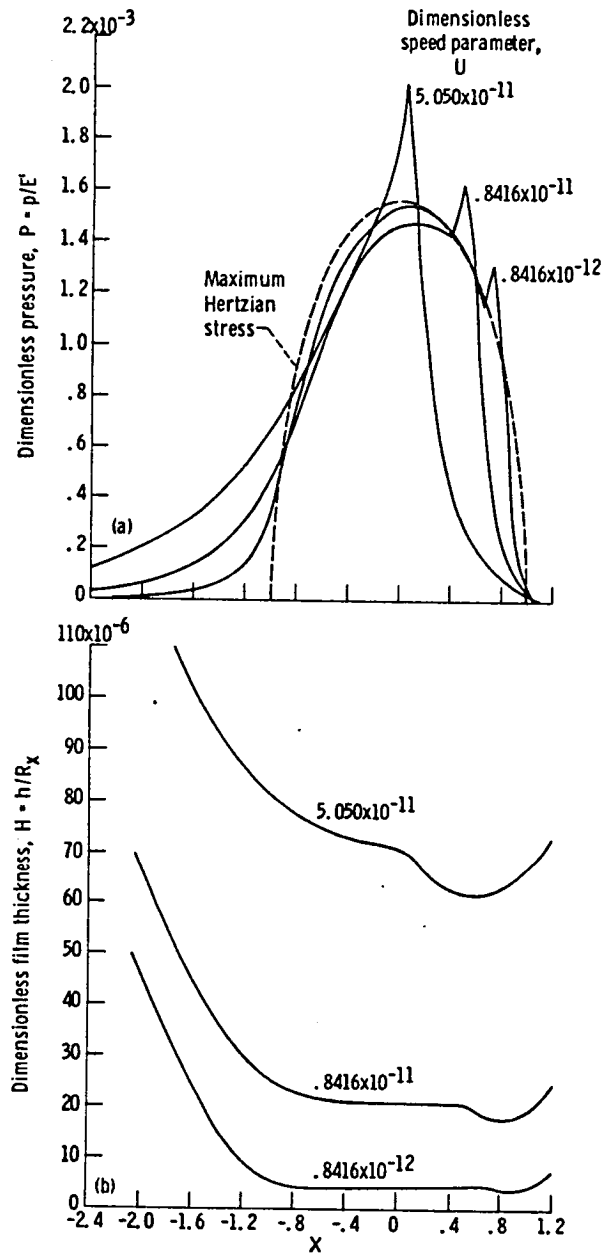
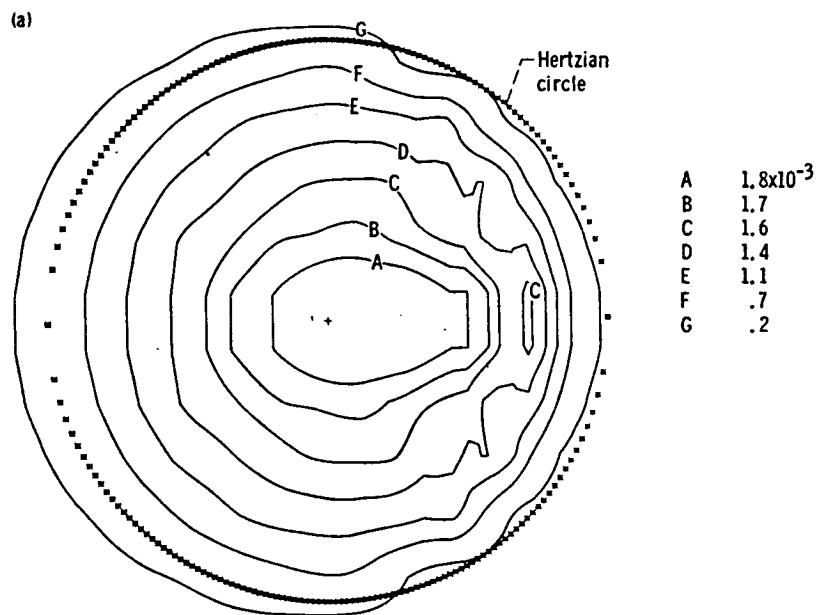
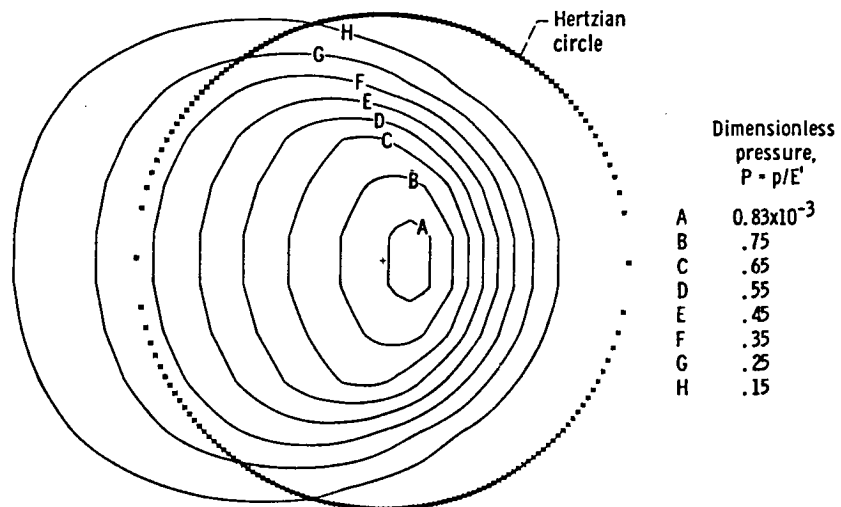
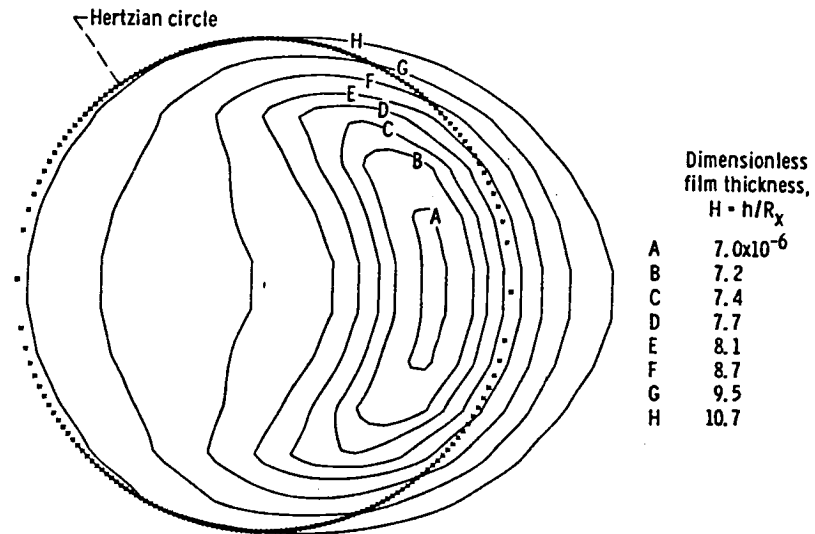


Figure 8.6. - Variation of dimensionless pressure and film thickness on X axis for three values of dimensionless speed parameter. The value of \bar{Y} is held fixed near axial center of contact. (a) Dimensionless pressure. (b) Dimensionless film thickness.

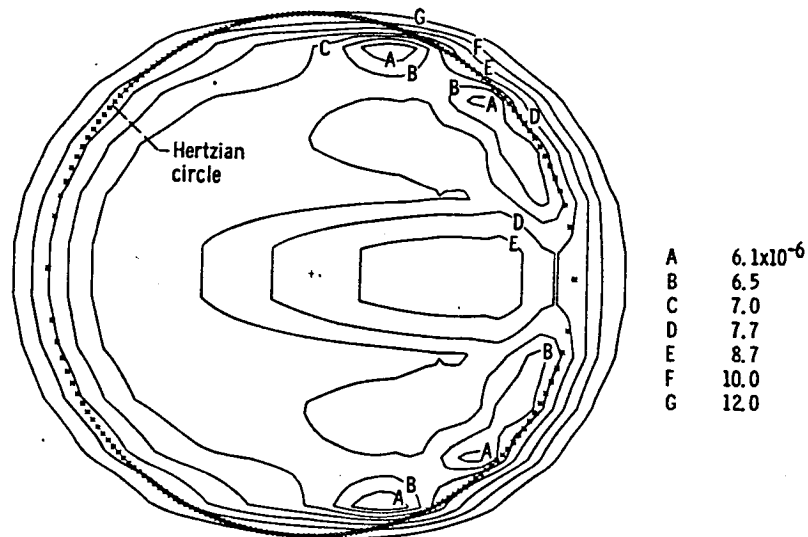


(b)

Figure 8.7. - Contour plots of dimensionless pressure for dimensionless load parameters W of 0.1106×10^{-6} and 0.1290×10^{-5} . The dimensionless parameters k , U , and G are held constant as defined in equation (8.17). (a) $W = 0.1106 \times 10^{-6}$. (b) $W = 0.1290 \times 10^{-5}$.



(a)



(b)

Figure 8.8 - Contour plots of dimensionless film thickness for dimensionless load parameters W of 0.1106×10^{-6} and 0.1290×10^{-5} . The dimensionless parameters k , U , and G are held constant as defined in equation (8.17). (a) $W = 0.1106 \times 10^{-6}$. (b) $W = 0.1290 \times 10^{-5}$.

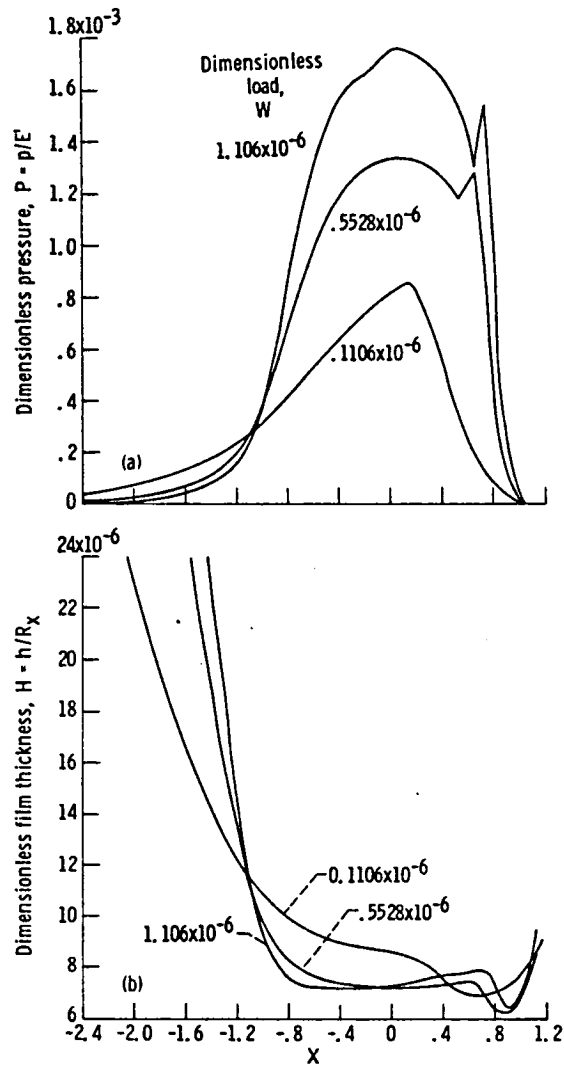


Figure 8.9. - Variation of dimensionless pressure and film thickness on X axis for three values of dimensionless load parameter. The value of Y is held fixed near axial center of contact. (a) Dimensionless pressure. (b) Dimensionless film thickness.

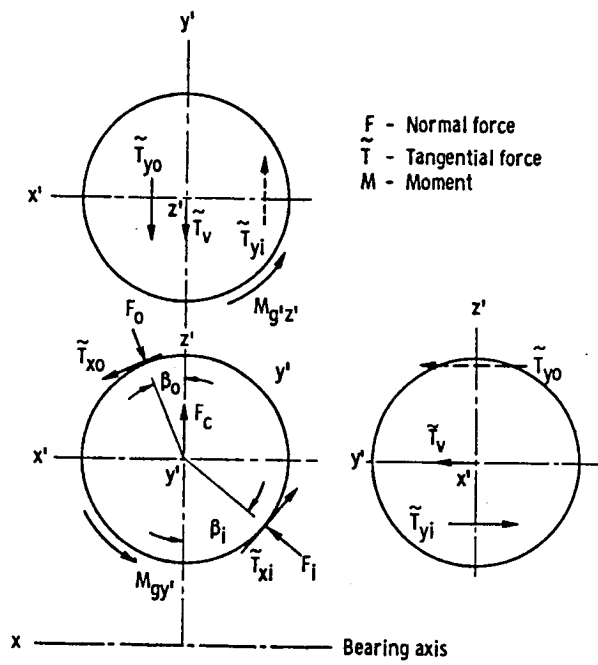


Figure 8.10. - Forces and moments acting on a ball.

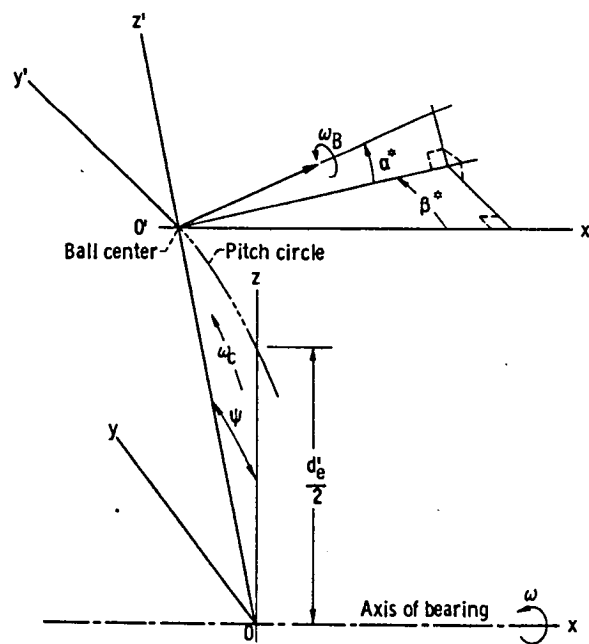


Figure 8.11. - Coordinate system.

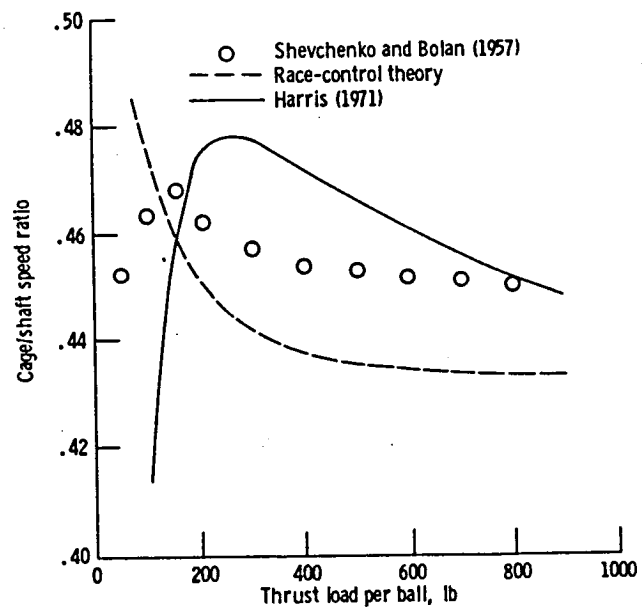


Figure 8.12. - Ratio of cage speed to shaft speed as function of thrust load per ball. Number of balls, 3; ball diameter, 1.125 in.; rotating speed, 9000 rpm.

1. Report No. NASA TM-81696		2. Government Accession No.		3. Recipient's Catalog No.	
4. Title and Subtitle ISOTHERMAL ELASTOHYDRODYNAMIC LUBRICATION OF POINT CONTACTS				5. Report Date October 1982	
				6. Performing Organization Code 505-32-42	
7. Author(s) Bernard J. Hamrock and Duncan Dowson				8. Performing Organization Report No. E-209	
9. Performing Organization Name and Address National Aeronautics and Space Administration Lewis Research Center Cleveland, Ohio 44135				10. Work Unit No.	
				11. Contract or Grant No.	
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D. C. 20546				13. Type of Report and Period Covered Technical Memorandum	
				14. Sponsoring Agency Code	
15. Supplementary Notes Bernard J. Hamrock, NASA Lewis Research Center, Cleveland, Ohio, and Duncan Dowson, Institute of Tribology, Department of Mechanical Engineering, The University of Leeds, Leeds, England. Published as Chapter 8 in Ball Bearing Lubrication by Bernard J. Hamrock and Duncan Dowson, John Wiley & Sons, Inc., September 1981.					
16. Abstract <p>Utilizing the theory developed by the authors in an earlier publication, the influence of the ellipticity parameter and the dimensionless speed, load, and materials parameters on minimum film thickness was investigated. The ellipticity parameter was varied from 1 (a ball-on-plate configuration) to 8 (a configuration approaching a line contact). The dimensionless speed parameter was varied over a range of nearly two orders of magnitude. Conditions corresponding to the use of solid materials of bronze, steel, and silicon nitride and lubricants of paraffinic and naphthenic mineral oils were considered in obtaining the exponent in the dimensionless materials parameter. Thirty-four different cases were used in obtaining the minimum-film-thickness formula $H_{min} = 3.63U^{0.68}G^{0.49}W^{-0.073}(1 - e^{-0.68k})^{0.64}$. A simplified expression for the ellipticity parameter was found where $k = 1.03(R_y/R_x)^{0.64}$. Contour plots were also shown which indicate in detail the pressure spike and two side lobes in which the minimum film thickness occurs. These theoretical solutions of film thickness have all the essential features of the previously reported experimental observations based upon optical interferometry.</p>					
17. Key Words (Suggested by Author(s)) Point contacts; Minimum film thickness; Ellipticity parameter; Dimensionless load, speed, and materials parameters			18. Distribution Statement Unclassified - unlimited STAR Category 37		
19. Security Classif. (of this report) Unclassified		20. Security Classif. (of this page) Unclassified		21. No. of Pages	
				22. Price*	

National Aeronautics and
Space Administration

Washington, D.C.
20546

Official Business

Penalty for Private Use, \$300

SPECIAL FOURTH CLASS MAIL
BOOK



Postage and Fees Paid
National Aeronautics and
Space Administration
NASA-451

NASA

POSTMASTER: If Undeliverable (Section 154
Postal Manual) Do Not Return
